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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

ANALYSIS OF STATICALLY INDETERMINATE TRUSSED STRUCTURES BY SUCCESSIVE APPROXIMATIONS

BY O. T. VOODHIGULA,¹ JUN. AM. SOC. C. E.

SYNOPSIS

In this paper the writer has attempted to simplify the classical method of analyzing statically indeterminate trussed structures, and to develop a physical concept whereby an analysis may be made rapidly by successive approximations.

INTRODUCTION

So far as the writer knows, there are two direct methods of successive approximation, applied to indeterminate trussed structures. Both methods were developed in England; the one by R. V. Southwell and the other by J. F. Baker, Assoc. M. Am. Soc. C. E., and A. J. Ockleston.² Due to the complicated nature of statically indeterminate trussed structures, the direct method of distributing stress becomes tedious. Professor Pippard states³ that "It is clear that less labor was involved in determining the forces in the members of the framework * * * by strain energy methods than by either of the successive approximation methods * * *." The method of work still seems to be the best. The only objection lies in the solution of the simultaneous equations since the labor involved increases rapidly with the number of equations.

METHOD

The structure is made statically determinate by cutting the redundant bars and replacing each of them with a pair of equal and opposite forces, as shown in Fig. 1. When the loads are applied, there will be relative movements at different cuts due to the distortions of the bars; the ends of the cut bars will move away from or toward each other, depending on the nature of the distortions. The relative movement of the two ends can be found by the method of

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by April 15, 1941.

¹ Municipal and Public Works, Bangkok, Thailand.

² "The Analysis of Engineering Structures," by A. J. S. Pippard, M. Am. Soc. C. E., and J. F. Baker, New York, 1936.

³ *Loc. cit.*, p. 108.

work. Let the relative movements due to the distortion of the statically determinate structure, of the two ends at a , b , and c , be δ_{a0} , δ_{b0} , δ_{c0} , respectively; thus

$$\delta_{a0} = \sum \frac{s u_a L}{A E} \dots \dots \dots (1a)$$

$$\delta_{b0} = \sum \frac{s u_b L}{A E} \dots \dots \dots (1b)$$

and

$$\delta_{c0} = \sum \frac{s u_c L}{A E} \dots \dots \dots (1c)$$

in which: s is the stress in each bar due to the applied load; u_a , u_b , and u_c are the stresses in each bar due to a unit resistance to movement at points a , b , and c , respectively; L = the length of the bar; A = the cross-sectional area of the bar; and E = the modulus of elasticity.

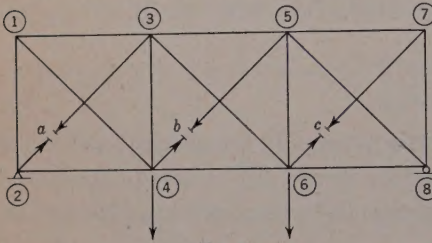


FIG. 1

The gaps must be closed by applying the forces X_a , X_b , X_c simultaneously. The relative movements at a , b , and c due to these three

forces must be equal to the relative movements δ_{a0} , δ_{b0} , and δ_{c0} caused by the loads on the base structure; that is:

$$-\sum \frac{s u_a L}{A E} = X_a \sum \frac{u_a u_a L}{A E} + X_b \sum \frac{u_a u_b L}{A E} + X_c \sum \frac{u_a u_c L}{A E} + \dots + X_n \sum \frac{u_a u_n L}{A E} \dots \dots \dots (2a)$$

$$-\sum \frac{s u_b L}{A E} = X_a \sum \frac{u_b u_a L}{A E} + X_b \sum \frac{u_b u_b L}{A E} + X_c \sum \frac{u_b u_c L}{A E} + \dots + X_n \sum \frac{u_b u_n L}{A E} \dots \dots \dots (2b)$$

and

$$-\sum \frac{s u_c L}{A E} = X_a \sum \frac{u_c u_a L}{A E} + X_b \sum \frac{u_c u_b L}{A E} + X_c \sum \frac{u_c u_c L}{A E} + \dots + X_n \sum \frac{u_c u_n L}{A E} \dots \dots \dots (2c)$$

or they may be written in the following forms:

$$-\delta_{a0} = X_a \delta_{aa} + X_b \delta_{ab} + X_c \delta_{ac} + \dots + X_n \delta_{an} \dots \dots \dots (3a)$$

$$-\delta_{b0} = X_a \delta_{ba} + X_b \delta_{bb} + X_c \delta_{bc} + \dots + X_n \delta_{bn} \dots \dots \dots (3b)$$

and

$$-\delta_{c0} = X_a \delta_{ca} + X_b \delta_{cb} + X_c \delta_{cc} + \dots + X_n \delta_{cn} \dots \dots \dots (3c)$$

In Eqs. 3 all the terms except X_a , X_b , and X_c are known or can be computed; δ_{a0} , δ_{b0} , and δ_{c0} are dependent on the loads, their positions, and the given

structure; and δ_{aa} , δ_{ab} , δ_{ac} , δ_{bb} , etc., are dependent on the given structure only and can be considered as the constants. This knowledge at once suggests that the equations can be solved by means of successive approximations. In Eq. 3a assume X_b and X_c equal to zero to obtain the approximate value of X_a . With X_a known and assuming $X_c = 0$, X_b can be computed from Eq. 3b. The approximate value of X_c can be solved from Eq. 3c. By repeating the foregoing process, successive values of X_a , X_b , and X_c will be obtained and these will approach the solution of Eqs. 3, if there is convergence. This is the solution of simultaneous equations by iteration.

In this problem a technique is needed in order to simplify the process of approximation to reveal the physical significance. Rearrange Eqs. 3 in the following forms:

$$X_a = -\frac{\delta_{a0}}{\delta_{aa}} - X_b \frac{\delta_{ab}}{\delta_{aa}} - X_c \frac{\delta_{ac}}{\delta_{aa}} \cdots (\text{etc.}) \dots \dots \dots (4a)$$

$$X_b = -\frac{\delta_{b0}}{\delta_{bb}} - X_a \frac{\delta_{ba}}{\delta_{bb}} - X_c \frac{\delta_{bc}}{\delta_{bb}} \cdots (\text{etc.}) \dots \dots \dots (4b)$$

and

$$X_c = -\frac{\delta_{c0}}{\delta_{cc}} - X_a \frac{\delta_{ca}}{\delta_{cc}} - X_b \frac{\delta_{cb}}{\delta_{cc}} \cdots (\text{etc.}) \dots \dots \dots (4c)$$

They can be still further simplified as follows:

$$X_a = -s_a' - X_b K_{ab} - X_c K_{ac} \cdots (\text{etc.}) \dots \dots \dots (5a)$$

$$X_b = -s_b' - X_a K_{ba} - X_c K_{bc} \cdots (\text{etc.}) \dots \dots \dots (5b)$$

and

$$X_c = -s_c' - X_a K_{ca} - X_b K_{cb} \cdots (\text{etc.}) \dots \dots \dots (5c)$$

The new notations may be defined as follows:

$s_a' = \frac{\delta_{a0}}{\delta_{aa}}$ = single redundant stress; that is, the stress in the redundant a when all the other redundants are not acting; and

$K_{ab} = \frac{\delta_{ab}}{\delta_{aa}}$ = stress in bar a due to a unit axial load in bar b ; it may be called a carry-over factor of stress of a due to a unit axial load acting in the redundant b .

Eqs. 5 are easier to follow than the previous equations, and the physical significance of every term is clear and enables one to visualize the stress distribution in the structure. It may be stated that the stress in any redundant is equal to the single redundant stress of the bar plus the carried over stresses from the other redundants. One may begin with the single redundant stresses, like fixed-end moments in the moment-distribution method,⁴ and then carry over the stresses from the other single redundants according to their carry-over factors. After the first approximation is complete, use the new values of the redundants and make the second approximation and so on. The convergence is very rapid as will be seen in the examples. This process of approximation is

⁴"Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, M. Am. Soc. C. E., Transactions, Am. Soc. C. E., Vol. 96 (1932), p. 1.

continued until the successive values of X become constant. In practical problems the approximations do not have to be extended so far; it is up to the designers to decide when to stop, depending on what degree of precision is required. This method is general and can also be extended to the external redundants.

The single redundant stresses s' can be computed readily by the method of work and so can the carry-over factors K . The computation of K is not tedious. Maxwell's law of reciprocal deflection is helpful for this purpose, and by inspection one can see what bars contribute to the deflection; and thus much unnecessary work can be avoided. After the constants K and s' are determined and tabulated, the computation is simple. The method can be best illustrated by examples.

A single redundant stress in the redundant a is expressed by

$$s_a' = \frac{\delta_{a0}}{\delta_{aa}} = \frac{\sum \frac{s u_a L}{A E}}{\sum \frac{u_a u_a L}{A E}} \dots \dots \dots (6)$$

in which: δ_{a0} and δ_{aa} are the relative movements of the cut a due, respectively, to the distortion of the statically determinate structure and an axial load of unity applied at the cut. The first subscript defines the position of movement or stress, and the second defines the position of the applied load. Note that Eq. 6 is dependent on the loads and the given structure.

The carry-over factor of the redundant a from redundant b is expressed as

$$K_{ab} = \frac{\delta_{ab}}{\delta_{aa}} = \frac{\sum \frac{u_a u_b L}{A E}}{\sum \frac{u_a u_a L}{A E}} \dots \dots \dots (7)$$

Unlike Eq. 6, this formula is independent of the loads.

TABLE 1.—COMPUTATION OF CONSTANTS, EXAMPLE 1

(a) PANEL 3						(b) PANEL 4 ^a				
Bar (Fig. 2)	$\frac{L}{A}$	s	u_a	$\frac{s u_a L}{A}$	$\frac{u_a^2 L}{A}$	Bar (Fig. 2)	s	u_b	$\frac{s u_b L}{A}$	$\frac{u_b^2 L}{A}$
4-6	5.39	-0.807	-0.628	+ 2.74	+ 2.13	6-8	-0.807	-0.628	+ 2.74	+ 2.13
5-7	5.97	-1.074	-0.628	- 4.02	+ 2.36	7-9	+0.538	-0.628	- 2.02	+ 2.36
4-5	18.15	+0.333	-0.778	- 4.70	+11.00	8-9	-0.333	-0.778	+ 4.70	+11.00
6-7	18.76	0	-0.778	0	+11.35	6-7	0	-0.778	0	+11.35
4-7	24.10	-0.428	+1.000	-10.30	+24.10	7-8	+0.428	+1.000	+10.30	+24.10
5-6	24.10	0	+1.000	0	+24.10	6-9	0	+1.000	0	+24.10
				-16.28 ^b	+75.04 ^c				+15.72 ^d	+75.04 ^e

^a L/A is the same for the bars in Panel 4 as in the bars of Panel 3. ^b $\delta_{a0} E$. ^c $\delta_{aa} E$ in Panel 3 and $\delta_{bb} E$ in Panel 4. ^d $\delta_{ba} E$.

EXAMPLE 1.—TWO REDUNDANTS

The relative movement of the cut a (Fig. 2(a)) depends on the distortions of the bars in panel 3 only, and similarly the relative movement of cut b is due to the distortions of the bars in panel 4.

The solution of Eqs. 8 is shown in Fig. 3, the correct values⁵ being $X_a = +0.254$; and $X_b = -0.248$.

TABLE 2.—COMPUTATION OF CONSTANTS, EXAMPLE 2

Bar ^a	PANELS ^a LEFT AND RIGHT SYMMETRICAL			PANEL LEFT		PANEL RIGHT		
	$\frac{L}{A}$	u_x	$\frac{u_x^2 L}{A}$	s	$\frac{s u_x L}{A}$	s	$\frac{s u_x L}{A}$	Bar ^a
PANEL ONE						PANEL EIGHT		
1-3	14.40	-0.60	+ 5.18	-1.5	+12.98	0	0	15-17
2-4	14.40	-0.60	+ 5.18	0	-0	+4.5	-38.90	16-18
1-2	17.45	-0.80	+11.15	-2.0	+27.90	0	0	17-18
3-4	32.00	-0.80	+20.05	-2.0	+51.20	+6.0	-153.60	15-16
1-4	19.20	+1.00	+19.20	+2.5	+48.00	0	0	17-16
3-2	19.20	+1.00	+19.20	0	+0	-7.5	-144.0	15-18
			+79.96 ^b		140.0 ^c		-336.50 ^c	
PANEL TWO						PANEL SEVEN		
3-5	8	-0.60	+ 2.88	-3.0	+14.40	-4.5	+ 21.60	13-15
4-6	8	-0.60	+ 2.88	+1.5	- 7.20	+9.0	- 43.20	14-16
3-4	32	-0.80	+20.05	-2.0	+51.20	+6.0	-153.60	15-16
5-6	32	-0.80	+20.05	-2.0	+51.20	+6.0	-153.60	13-14
3-6	24	+1.00	+24.00	+2.5	+60.00	0	0	15-14
5-4	24	+1.00	+24.00	0	0	-7.5	-180.00	13-16
			+93.86 ^b		169.60 ^c		-508.80 ^c	
PANEL THREE						PANEL SIX		
5-7	5.76	-0.60	+ 2.08	-4.5	+ 15.57	-9.0	+ 31.10	11-13
6-8	5.76	-0.60	+ 2.08	+3.0	-10.38	+7.5	- 25.90	12-14
5-6	32.00	-0.80	+ 20.05	-2.0	+ 51.20	+6.0	-153.60	13-14
7-8	32.00	-0.80	+ 20.05	-2.0	+ 51.20	-2.0	+ 51.20	11-12
5-6	34.30	+1.00	+ 34.30	+2.5	+ 85.80	0	0	13-12
7-6	34.30	+1.00	+ 34.30	0	0	+2.5	+ 85.80	11-14
			+112.86 ^b		+193.39 ^c		- 11.30 ^c	
PANEL FOUR						PANEL FIVE		
7-9	4.97	-0.60	+ 1.79	-6.0	+ 17.90	-7.5	+ 22.40	9-11
8-10	4.97	-0.60	+ 1.79	+4.5	- 13.40	+6.0	- 17.90	10-12
7-8	32.00	-0.80	+ 20.05	-2.0	+ 51.20	-2.0	+ 51.20	11-12
9-10	32.00	-0.80	+ 20.05	-2.0	+ 51.20	-2.0	+ 51.20	9-10
7-10	40.00	+1.00	+ 40.00	+2.5	+100.00	0	0	11-10
9-8	40.00	+1.00	+ 40.00	0	0	+2.5	+100.00	9-12
			+123.68 ^b		+206.90 ^c		+206.90 ^c	

^a See Fig. 4, $u_a = u_b$, $u_b = u_g$, $u_c = u_f$, $u_d = u_e$; also, $x = a, b, c$, etc., according to panel. ^b $\delta_{aa} E$, $\delta_{bb} E$, $\delta_{cc} E$, etc. ^c $\delta_{aa} E$, $\delta_{bb} E$, $\delta_{cc} E$, etc.

EXAMPLE 2.—EIGHT REDUNDANTS

Similarly to Example 1, constants for substitution in Eqs. 5 are computed as in Table 2, for use in determining the single redundant stresses

⁵ "Theory of Statically Indeterminate Structures," by W. M. Fife and J. B. Wilbur, Assoc. Members, Am. Soc. C. E., McGraw-Hill Book Co., Inc., 1937, p. 166.

and

$$K_{hg} = \frac{\delta_{hg}}{\delta_{hh}} = \frac{(-20.45)}{79.96} = -0.256.$$

The solution for the redundant stresses by Eqs. 8 is shown in Table 3. This is a classic problem solved by Messrs. Johnson, Bryan, and Turneure.⁶

TABLE 3.—COMPUTATIONS OF
(In s_x' , the Subscript $x = a$,

Cycle No.	PANEL ONE		PANEL TWO			PANEL THREE			PANEL FOUR		
	$K_{ab} X_b$	X_a	$K_{ba} X_a$	$K_{bc} X_c$	X_b	$K_{cb} X_b$	$K_{cd} X_d$	X_c	$K_{dc} X_c$	$K_{de} X_e$	X_d
s_x'	-1.750		-1.810			-1.715			-1.675		
K	-0.256		-0.218			-0.181			-0.165		
1	+0.463	-1.750			-1.810			-1.715			-1.675
2	+0.292	-1.347	+0.293	+0.374	-1.143	+0.207	+0.303	-1.205	+0.199	+0.278	-1.198
3	+0.314	-1.458	+0.318	+0.263	-1.229	+0.222	+0.217	-1.276	+0.210	+0.246	-1.219
		-1.436	+0.313	+0.278	-1.219	+0.221	+0.221	-1.273	+0.210	+0.226	-1.239
		-1.44			-1.21			-1.25			-1.23

Moving Loads.—In applying the method of successive approximations, it is obvious that determining the carry-over factors and the single redundants requires most of the labor; the process of approximations requires little effort. The carry-over factors, once found, can be used for any loadings. The single redundants change with changes in loads.

EXAMPLE 3.—EXTENSION OF MÜLLER-BRESLAU'S PRINCIPLE TO BAR STRESS

The Müller-Breslau principle is probably the best means of plotting influence lines; the principle can also be extended to a bar stress. When the influence line of any bar is required, introduce a pair of axial forces at the two ends of the bar so that a unit distortion is produced. The deflected load line of the structure is the influence line for the stress of that bar.

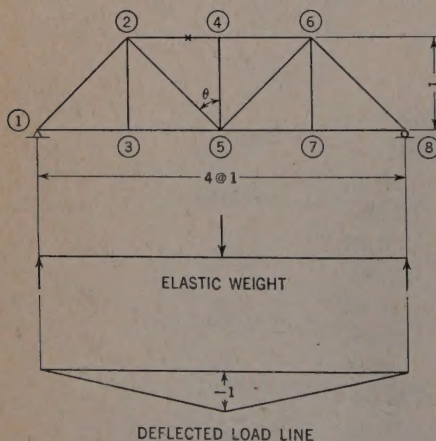


Fig. 5

Suppose the influence line of the stress in the bar 2-4, Fig. 5, is required. Introduce a unit negative distortion in bar 2-4; in this case the distortion of the bar does not affect the stress in the other bars. The angle change $\Delta\theta = \frac{1}{1} = 1$; the moment curve of the imaginary beam is the deflected load line and it is the influence line for the stress in the

⁶ "Modern Framed Structures," by C. W. Bryan, Jr., M. Am. Soc. C. E., F. E. Turneure, Hon. M. Am. Soc. C. E., and the late J. B. Johnson, M. Am. Soc. C. E., Pt. II, 10th Ed. (1929), p. 310.

bar 2-4. This can be verified readily by statics.

The following conventions must be observed: A negative angle change is an elastic load acting downward. The positive moment curve must be drawn below the reference line, because positive moment causes the beam to deflect downward. It does not matter whether negative distortion or positive dis-

REDUNDANT STRESSES, EXAMPLE 2
b, c, Etc., According to the Panel)

PANEL FIVE			PANEL SIX			PANEL SEVEN			PANEL EIGHT		Cycle No.
$K_{ed} X_d$	$K_{ef} X_f$	X_e	$K_{fe} X_e$	$K_{fg} X_g$	X_f	$K_{gf} X_f$	$K_{gh} X_h$	X_g	$K_{hg} X_g$	X_h	
-1.675			+0.100			+5.425			+4.210		s_d'
-0.165			-0.181			-0.218			-0.256		K
+0.198	-0.017	-1.675	+0.271	-0.982	+0.100	+0.133	-0.916	+5.425	-1.189	+4.210	1 2 3
+0.201	+0.101	-1.494	+0.245	-0.840	-0.611	+0.108	-0.658	+4.642	-1.250	+3.021	
+0.204	+0.082	-1.373	+0.251	-0.883	-0.495	+0.116	-0.645	+4.875	-1.270	+2.960	
		-1.388			-0.532			+4.896		+2.940	
		-1.37			-0.53			+4.86		+2.93	

tortion is applied to the bar, the sign of stress caused by a load can be determined easily by inspection. Suppose the load is applied at point 7 of the equivalent beam; it is obvious that the beam will deflect down farther and it means that the assumed compression stress is correct. If positive distortion is applied in the bar 2-4, the deflected load line is above the reference line. In this case, when the load is applied at point 7, the load tends to decrease the deflection which means that the sign is opposite to what it was assumed to be.

EXAMPLE 4.—DETERMINATION OF THE INFLUENCE LINE OF THE SINGLE REDUNDANT STRESS s_d' OF THE TRUSS IN EXAMPLE 2

When a distortion is applied to a single redundant, the problem is not as simple as in the previous case, because stresses will be set up in some other bars. However, it can be solved conveniently by the method of elastic weight. The constants for substitution in Eq. 5 are determined by setting up a computation form similar to Table 2, Panel 1, from which $s_d' = \frac{s_{ud} L}{A} = \frac{-83.68}{123.68} = 0.677$.

Stresses.—The force required to shorten bar 8-9, Fig. 6(a), 1 in. is $X = \frac{1.00}{0.677} \times \frac{12 \times 30,000,000}{40 \times 12} = 1,107,000$ lb.

Angle Changes.—Formulas for the angle changes (see Fig. 6(d)) are:

$$\Delta\gamma = \frac{1}{E} [(f_e - f_b) \cot \alpha + (f_e - f_a) \cot \beta] \dots\dots\dots (9a)$$

$$\Delta\beta = \frac{1}{E} [(f_b - f_a) \cot \alpha + (f_b - f_e) \cot \gamma] \dots\dots\dots (9b)$$

and

$$\Delta\alpha = \frac{1}{E} [(f_a - f_e) \cot \beta + (f_a - f_b) \cot \gamma] \dots\dots\dots (9c)$$

The solution of Eqs. 9, applied to Fig. 6(e), is as follows:

Joint 8—

$$\Delta\theta_1 =$$

0

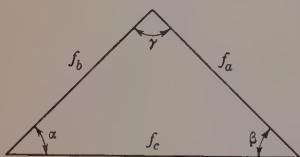
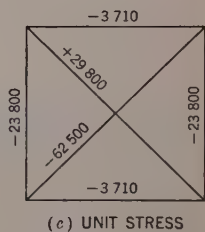
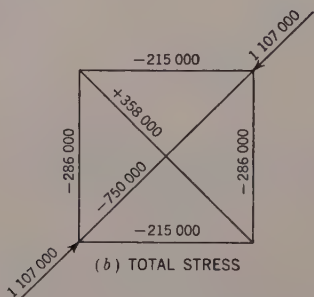
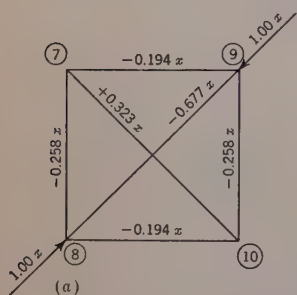
$$\Delta\theta_2 = \frac{1}{E} \left[(+29,800 + 23,800) \frac{32}{24} + (+29,800 + 3,710) \frac{24}{32} \right]$$

$$= \frac{1}{E} [+71,500 + 25,100]$$

$$= \frac{+96,600}{E}$$

Total angle change at joint 8, Fig. 6(e)

$$= \frac{+96,600}{E}$$



(d)

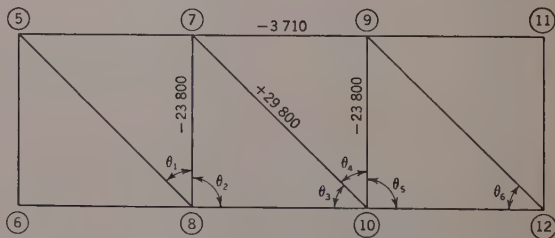


FIG. 6

Joint 10—

$$\Delta\theta_3 = \frac{1}{E} \left[(-23,800 - 29,800) \frac{32}{24} + (-23,800 + 3,710) 0 \right]$$

$$= \frac{-71,500}{E}$$

$$\Delta\theta_4 = \frac{1}{E} \left[(-3,710 + 23,800) 0 + (-3,710 + 29,800) \frac{24}{32} \right]$$

$$= \frac{-25,100}{E}$$

$$\Delta\theta_5 = \frac{1}{E} (0 + 23,800) \frac{32}{24}$$

$$= \frac{+31,700}{E}$$

Total angle change at joint 10

$$= \frac{-64,900}{E}$$

Joint 12—

$$\Delta\theta_6 = \frac{1}{E} (-23,800 - 0) \frac{32}{24}$$

$$= \frac{-31,700}{E}$$

Total angle change at joint 12

$$= \frac{-31,700}{E}$$

Influence ordinates are then computed, thus (see Fig. 7): For a unit load at joint 8, $s_d' = \frac{+16,100 \times 3 \times 24 \times 12}{30,000,000} \times 0.677 = +0.314$; for a unit load at joint 10, $s_d' = \frac{(+16,100 \times 4 - 96,600 \times 1) 24 \times 12 \times 0.677}{30,000,000} = -0.210$; and, for a unit load at joint 12, $s_d' = \frac{-16,100 \times 3 \times 24 \times 12}{30,000,000} \times 0.677 = -0.314$. To check these results assume a load of 8 units at joint 14, Fig. 7: By computation, $s_d' = -1.675$; and from the influence line, $s_d' = -0.314 \times \frac{2}{3} \times 8$

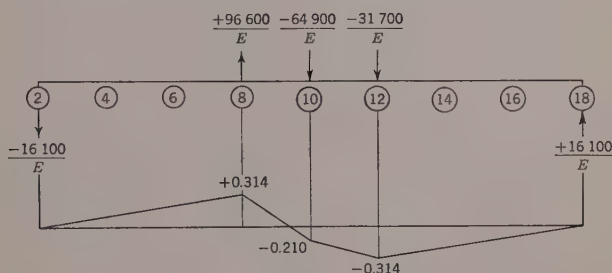
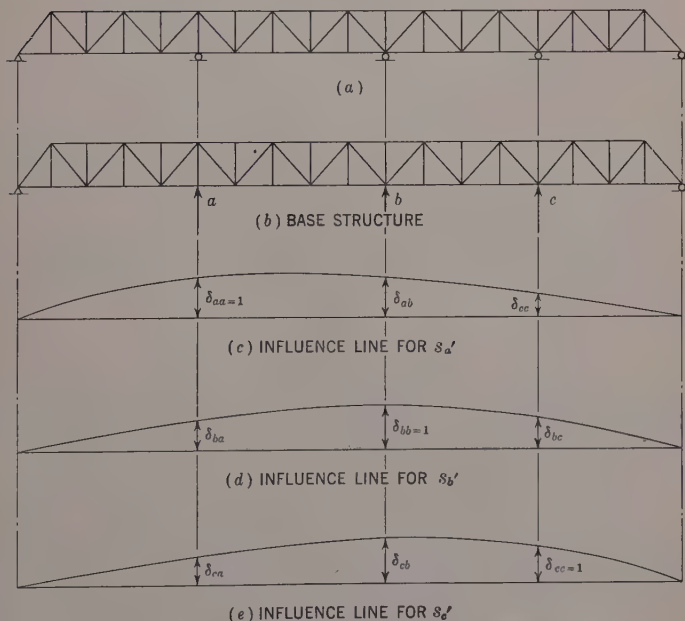
FIG. 7.—INFLUENCE LINE FOR s_d' 

FIG. 8

$= -1.675$, which checks. Note that the deflection of the imaginary beam, which is equal to the moment of the beam at that point, is equal to the relative movement of the joints 8 and 9. The actual stress set up in the bar 8-9 is only 0.677 of the force applied.

EXTERNAL REDUNDANTS

The same principle can be applied to external redundants. Single redundants and carry-over factors must be found for the process of approximations. In the case of external redundant structure shown in Fig. 8, it is advantageous to draw the influence lines for the single redundants, because the carry-over factors can be taken directly from the influence lines.

For example (see Eqs. 5):

From Fig. 8(c)—

$$K_{ab} = \frac{\delta_{ab}}{\delta_{aa}} = \frac{\delta_{ab}}{1} \dots \dots \dots (10a)$$

and

$$K_{ac} = \frac{\delta_{ac}}{\delta_{aa}} = \frac{\delta_{ac}}{1} \dots \dots \dots (10b)$$

from Fig. 8(d)—

$$K_{ba} = \frac{\delta_{ba}}{\delta_{bb}} = \frac{\delta_{ba}}{1} \dots \dots \dots (10c)$$

and

$$K_{bc} = \frac{\delta_{bc}}{\delta_{bb}} = \frac{\delta_{bc}}{1} \dots \dots \dots (10d)$$

and from Fig. 8(e)—

$$K_{ca} = \frac{\delta_{ca}}{\delta_{cc}} = \frac{\delta_{ca}}{1} \dots \dots \dots (10e)$$

and

$$K_{cb} = \frac{\delta_{cb}}{\delta_{cc}} = \frac{\delta_{cb}}{1} \dots \dots \dots (10f)$$

CONCLUSION

The method of successive approximations, as presented herein, should be useful to structural analysts in effecting a saving of time and a better understanding of the structure. For practical purposes the slide-rule may be used without the fear of accumulating errors as in the solution of simultaneous equations.

ACKNOWLEDGMENT

The writer expresses his appreciation to Albert Haertlein, M. Am. Soc. C. E., for guidance in the preparation of this paper.

LABORATORY INVESTIGATIONS OF SOILS AT FLUSHING MEADOW PARK

BY DONALD M. BURMISTER,¹ ASSOC. M. AM. SOC. C. E.

SYNOPSIS

Laboratory investigations made to determine the general suitability of the Flushing Meadow in New York, N. Y., for development as a permanent park and for supporting the temporary and permanent structures of the World's Fair, are described in this paper. The problems encountered in the reclamation and development of the site arise from the peculiar nature of the soils of the meadow, which are quite typical of such tidal marsh deposits. The tests included routine studies to determine the general physical character, and consolidation and shear tests to determine the behavior characteristics of the materials encountered. They were made primarily to furnish information on what could be done and how it should be done. The tests and their interpretation are discussed in relation to these problems, and certain suggestions are made as to the correlation of test results which permit the use of data obtained from the simpler tests as a basis for predicting the more general behavior of soils.

INTRODUCTION

The laboratory investigations for the development of the Flushing Meadow Park site for the New York World's Fair were interesting in many respects, not only because of the practical use made of the results of soil tests, but also because of the interesting nature of the problems encountered. The first series of tests was made during the latter part of 1935 for a preliminary report, by the consulting foundation engineers, on the suitability of the Flushing Meadow for development as a permanent park and for supporting the permanent and temporary structures of the World's Fair,² and for a later report by the same engineers during 1936 for certain permanent structures.

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by May 15, 1941.

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² "The Application of Soil Mechanics in Building the New York World's Fair," by George L. Freeman M. Am. Soc. C. E., Hamilton Gray, Jun. Am. Soc. C. E., and George W. Glick, *Civil Engineering*, October 1940, p. 649; see also "Soil Survey of the Flushing Meadow Park Site, Long Island, New York," by George L. Freeman, *Paper No. C-2, Proceedings, International Conference on Soil Mechanics and Foundation Eng., Harvard Univ., Cambridge, Mass., 1936, Vol. I, p. 25.*

The Flushing Meadow was a low-lying tidal marsh, only about 2 ft above mean tide, consisting of deposits of very soft silty clays extending to a depth of 60 ft or more in some places, and underlaid by sands and gravels. The materials of the deposit were sampled by forty-eight undisturbed samples from twelve borings, 3 in. in diameter, which were tested (see Table 1) primarily to

TABLE 1.—SCHEDULE OF SOIL TESTS, FLUSHING MEADOW PARK

Series	Borings	Samples	Routine tests	Consolidation tests	Shear tests
Preliminary report.....	12	48	48	20	19
Permanent structures..	25	70	70	19	48

furnish information on the nature of the problems to be encountered, and to serve as a guide for making recommendations as to:

- (1) Methods of filling and grading the site, stable slopes, etc., to prevent lateral subsurface soil movements;
- (2) The quantity of fill material required and finished grade elevations to compensate for the expected settlement due to the weight of the fill; and
- (3) The allowable foundation pressures in different parts of the area and the settlement to be expected when the structures were erected.

In addition, seventy undisturbed samples from twenty-five borings, 3 in. in diameter, taken at the proposed sites of permanent structures, were tested to furnish detailed information as to the foundation conditions and for use in selecting and designing the most satisfactory and economical types of foundations for the different types of structures. A more detailed study and analysis of the results of the soil tests than was possible at the time, together with the results of subsequent researches on the Flushing Meadow soils, are herewith offered to complete the information on the development of the Flushing Meadow Park site.

PHYSICAL CHARACTERISTICS

Routine tests were made on all samples for the purpose of classifying the soils. These tests included: Natural moisture content, grain-size analysis, liquid, plastic and shrinkage limits, specific gravity, loss on ignition, and a routine squeeze test.

These routine tests gave information on the general character of the soils encountered and their uniformity or variability. Furthermore, they showed whether the Flushing Meadow soils had regular characteristics³ usually associated with materials of their grain-size distribution, or whether they had unusual or peculiar characteristics that would be expected to influence their behavior greatly. For similar grain-size distributions, soils having regular characteristics have been found to possess similar properties. The peculiar nature of the Flushing Meadow soils is shown by the extremely high values of the liquid

³"The Physical Characteristics of Soils (with Special Reference to Earth Structures," by Donald M. Burmister, *Bulletin No. 6*, Dept. of Civ. Eng., Columbia Univ., New York, N. Y.

limit and plasticity index in relation to those for regular soils. These characteristics are due inherently to the mineralogical and chemical nature of the true clay fraction of the soil.

Microscopic studies and X-ray diffraction patterns show that the clay minerals consist chiefly of kaolinite and hydromica. The hydromica clay minerals, which are an intermediate alteration product of a scale-like crystalline structure formed by the weathering of feldspathic rocks to kaolinite, and the high organic content are responsible for the unusual character of the Flushing Meadow soil.

The natural moisture content is characteristically high for these materials, as shown in Fig. 1, being about 100% by dry weight, or more. A striking feature shown by these tests is the fact that the moisture content is practically uniform below the meadow mat down to the underlying sand and gravel layers, and about equal to the liquid limit, a condition which seems to be characteristic of such deposits. These facts explain why this material is so extremely soft in character, and indicate that any serious disturbance would tend to reduce it to consistencies approaching the liquid-limit condition. Although the meadow mat had still higher moisture contents of 200% or more, it had considerable strength because of the strong root system, a fact that made possible, and greatly facilitated, the grading operations. A marked peculiarity of samples from certain areas of the Flushing Meadow was the swelling properties they exhibited, due to the release of dissolved gases (probably marsh gas).

A most important property of the silty clay deposit is its natural structure, which gives the undisturbed material a much higher strength than when remolded or disturbed. It is important, therefore, to obtain preliminary information on the structure and consistency of all samples. The small cylinder test (or cube test) has been used for such purposes, on medium to stiff clays, by Karl Terzaghi,⁴ M. Am. Soc. C. E., and by others. A simplified small squeeze test device^{5, 6} was developed, for the very soft materials encountered in the Flushing Meadow, to obtain preliminary information on the structural capacity of these materials. A small sampling tube is used to cut a cylindrical specimen 2 sq cm in area by 1 cm high. The average final diameter, in centimeters, of the squeeze specimen, after applying a 2-kg load, is taken as the squeeze test value.

The structure of the clay is indicated in the soil profile of Fig. 1 by plotting the undisturbed squeeze test diameter, which is determined primarily by structure, and the remolded value, which is considerably greater and is influenced largely by moisture content.

It is interesting to note that the consistency of the material, as indicated by the undisturbed squeeze test value, does not increase appreciably with depth for boring No. 17, whereas for boring No. 2 the consistency of the upper layers

⁴"Determination of Consistency of Soils by Means of Penetration Tests," by Karl Terzaghi, *Public Roads*, February, 1927, Vol. 7, p. 240.

⁵"Squeeze Test for Integrity of Soil Samples," by Donald M. Burmister, *Engineering News-Record*, April 22, 1937, p. 588.

⁶"A New Method for Determining the Relative Consistency of Soils," by Donald M. Burmister, *Paper No. A-13, Proceedings, International Conference on Soil Mechanics and Foundation Eng.*, Harvard Univ., Cambridge, Mass., 1936, Vol. II, p. 43.

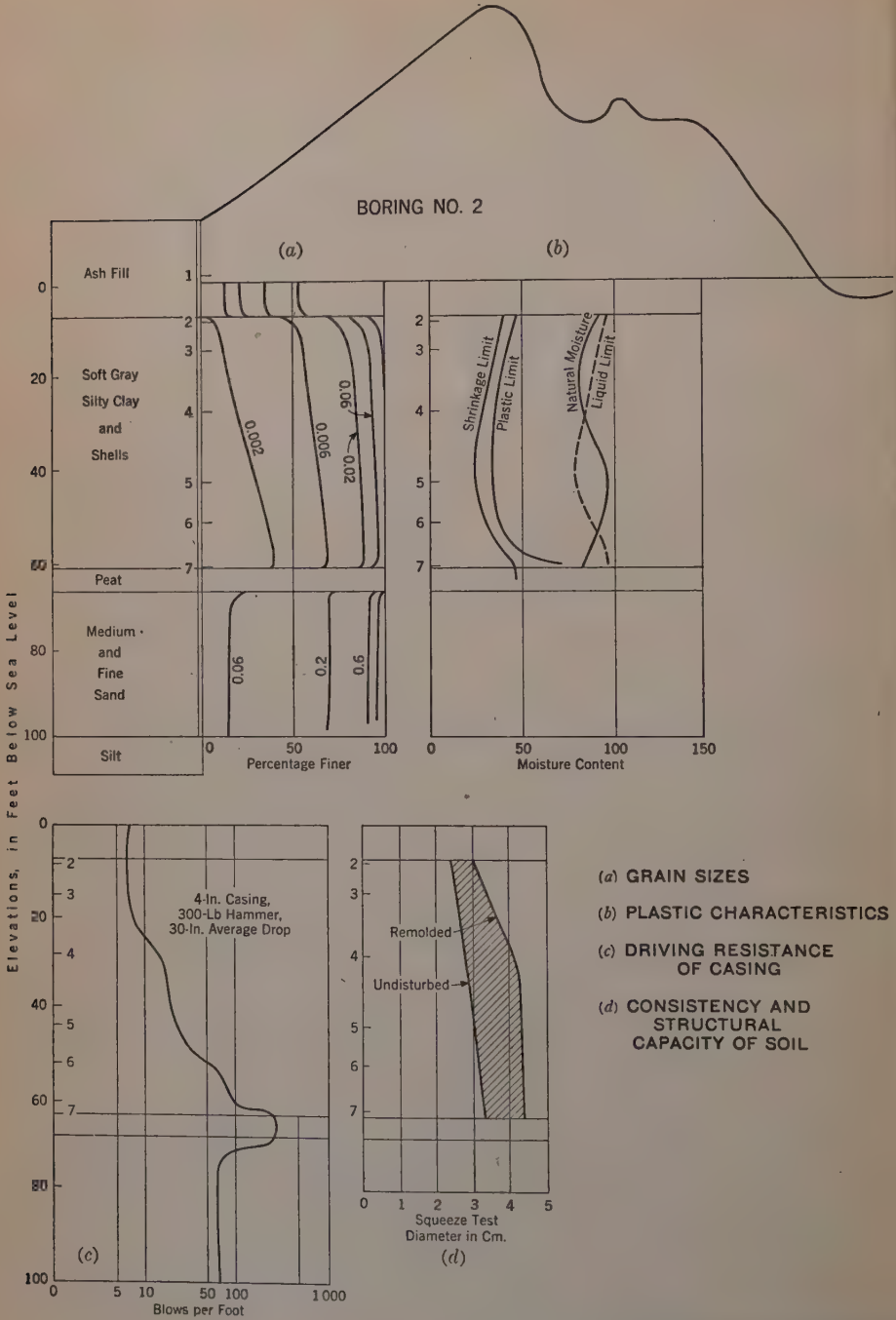
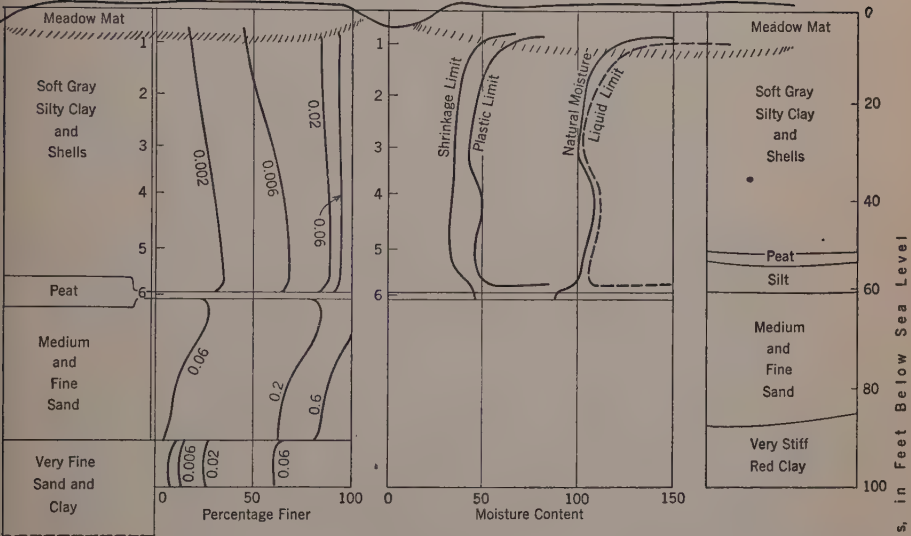


FIG. 1.—PHYSICAL CHARACTERISTICS

BORING NO. 17

(a)

(b)

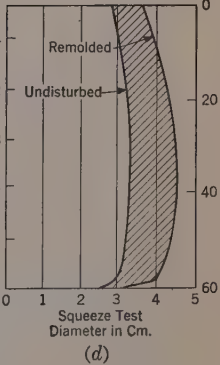
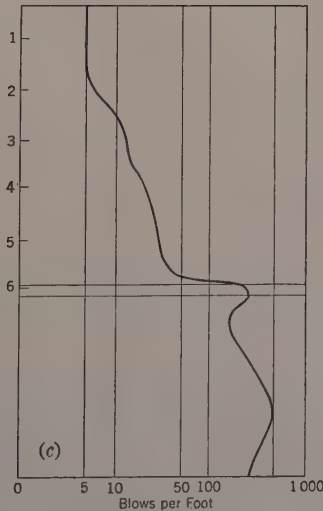


(a) GRAIN SIZES

(b) PLASTIC CHARACTERISTICS

(c) DRIVING RESISTANCE
OF CASING

(d) CONSISTENCY AND
STRUCTURAL
CAPACITY OF SOIL



directly beneath the ash dump is much stiffer as a result of the consolidation of the material under the weight of the ash fill.

The structural capacity of the soil is indicated by the numerical difference or spread between the squeeze test diameters for the undisturbed and remolded states, about one centimeter indicating a fairly undisturbed and representative sample for these soft Flushing Meadow soils. Correlations with shearing tests on these materials showed that a spread of one centimeter represented a ratio of 6 : 1 between the shearing strength in the undisturbed and remolded states. A comparison of the squeeze test diameters in the remolded state in Fig. 1 with the values obtained for the liquid limit and plastic limit conditions of remolded materials, respectively 3.70 and 1.60 cm, indicates how serious would be the effect of any disturbance to the natural structure of these silty clays with the consequent loss of shearing strength. The shearing strength at the liquid limit is approximately 0.02 kg per sq cm. The routine squeeze test may be used in the laboratory for selecting the more representative and least disturbed samples for the consolidation and shear tests, and also has possibilities in the field for control of sampling operations.

The extremely soft consistency of the materials encountered in each boring is also clearly indicated by typical driving records in Fig. 1. The presence of the underlying sand and gravel layers, together with the relative compactness of these deposits, is shown by the marked increase in driving resistance of the casing.

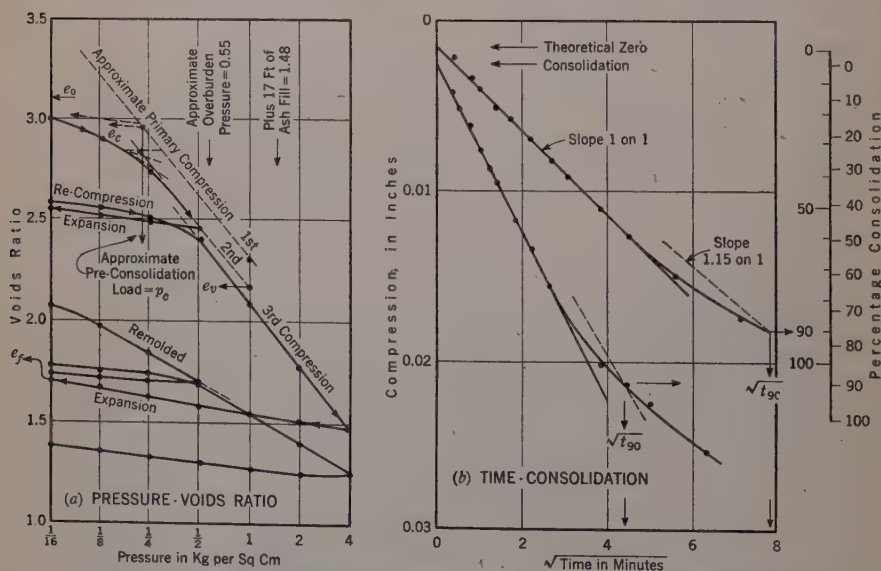


FIG. 2.—TYPICAL CONSOLIDATION CHARACTERISTICS OF THE FLUSHING MEADOW SOILS

CONSOLIDATION CHARACTERISTICS

The consolidation characteristics of the Flushing Meadow deposits were obtained from consolidation tests, which indicated that settlements of considerable magnitude were to be expected from the consolidation of the soft material under the weight of the fill, and of the Fair structures.

Typical pressure-voids ratio and time-consolidation curves are shown in Fig. 2. The subscript *v* denotes compression or volume-change phenomena in general. It is applicable to all materials, whether simple compression takes place, as in permeable, cohesionless soil; or whether consolidation takes place, as in saturated plastic clays. The subscript *c* denotes phenomena specifically related to consolidation. The consolidation characteristics required for settlement analyses are as follows:

- (1) The voids ratio *e_v* at a pressure of *p'* equal to 1 kg per sq cm;
- (2) The compression index *c_v* (slope of the pressure-voids ratio curve on a semi-log plot);
- (3) The pre-consolidation load *p_c*, as defined by Arthur Casagrande,⁷ Assoc. M. Am. Soc. C. E.;
- (4) The coefficient of consolidation *c_c*, as determined either by the Casagrande⁸ or the Taylor method.⁹

According to the Taylor diagram (Fig. 2) these coefficients are:

Item	Undisturbed	Remolded
2 <i>h_c</i> , in cm.	0.405	0.497
√ <i>t₉₀</i> , in min.	4.4	7.8
<i>t₉₀</i> , in min.	19.4	60.9
<i>c_c</i> , in sq cm per min.	0.0018	0.00078
Load increment, in kg per sq cm.	¼ to ½	¼ to ½

A comparison of consolidation coefficients is afforded by reference to Table 2.

TABLE 2.—CHARACTERISTICS OF MATERIAL IN BORING NO. 9, SAMPLE 5 (DEPTH = 52 FT), AT BRIDGE C ACROSS HORACE HARDING BOULEVARD

CONSOLIDATION CHARACTERISTICS				PHYSICAL CHARACTERISTICS	
Item	Symbol	Undis- turbed	Re- molded	Definition	Value
Voids ratio at pre-consolidation load.	<i>e_e</i>	2.96		Liquid limit	93.0
Voids ratio at <i>p'</i> = 1 kg per sq cm.	<i>e_v</i>	2.17	1.55	Plastic limit	42.2
Compression index.	<i>c_v</i>	0.361	0.216	Plasticity index	50.8
Coefficient of compressibility (¼ to ½ kg).	<i>c_a</i>	1.25	0.56	Specific gravity	2.73
Coefficient of consolidation in sq cm per min. .	<i>c_c</i>	0.0018	0.00078	Squeeze Test:	
				Undisturbed	2.9 ^a
				Remolded	4.2 ^a

^a Squeeze test units are centimeters.

DEFINING EQUATIONS

Two defining equations used in this paper are:

$$e = e_v - c_v \log_e \frac{p}{p'} \dots \dots \dots (1)$$

⁷ "The Determination of the Pre-Consolidation Load and Its Practical Significance," by Arthur Casagrande, *Paper No. D-34, Proceedings, International Conference on Soil Mechanics and Foundation Eng., Harvard Univ., Cambridge, Mass., 1936, Vol. III, p. 60.*
⁸ "New Facts in Soil Mechanics from the Research Laboratories," by Arthur Casagrande, *Engineering News-Record*, September 5, 1935, p. 320.
⁹ "Improved Soil Testing Methods," by Glennon Gilboy, Assoc. M. Am. Soc. C. E., *loc. cit.*, May 21, 1936, p. 732.

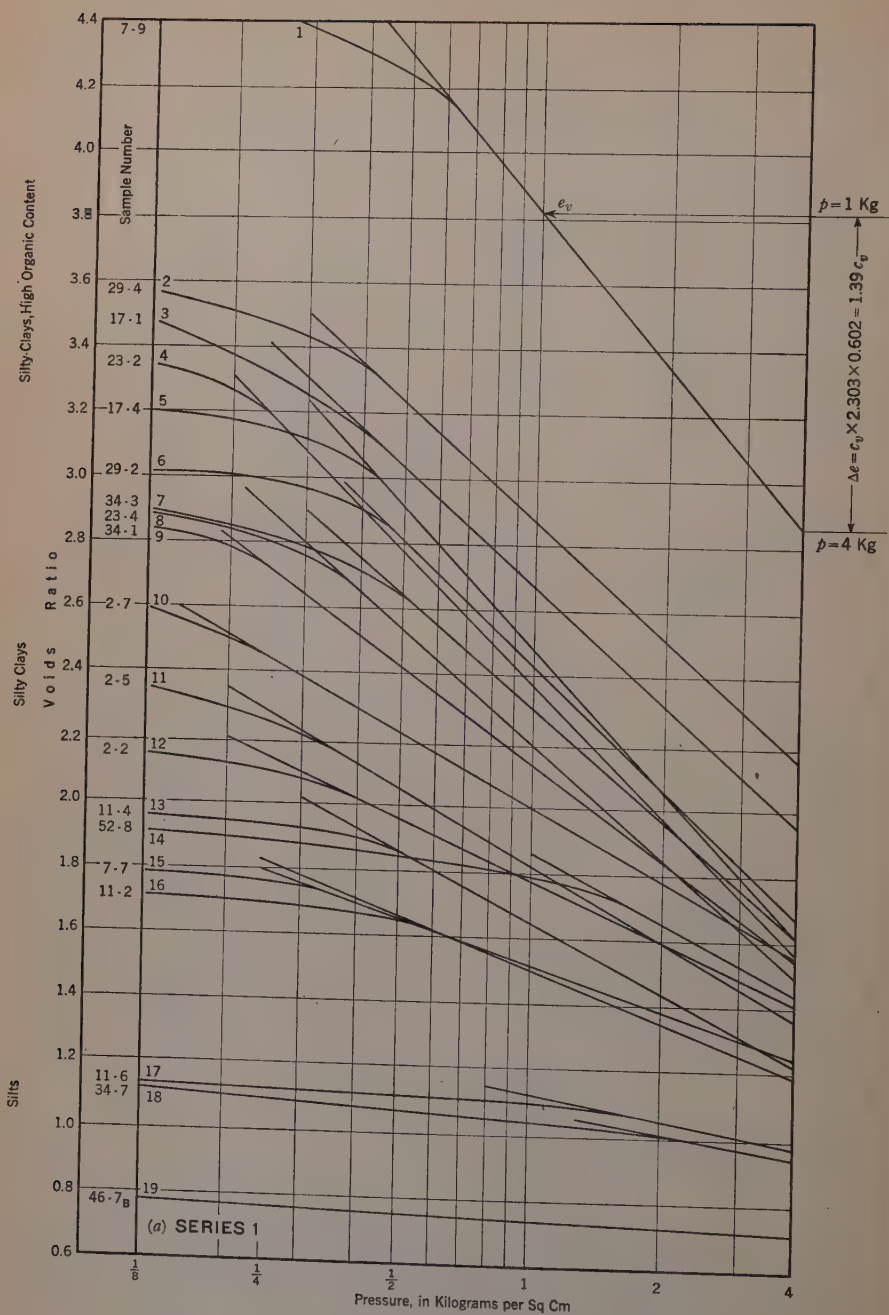


FIG. 3.—PRESSURE-VOIDS RATIO CURVES, FLUSHING MEADOW SOILS

and, total settlement by the summation method,

$$S = \sum \Delta h \frac{e_1 - e_2}{1 + e_1} \dots \dots \dots (2)$$

in which $(e_1 - e_2)$ corresponds to the initial and final pressures, p_1 and p_2 , respectively, at the center of the soil layer, the thickness being Δh .

The organic silty clays of the Flushing Meadow deposits are characterized by steep pressure-voids ratio curves, high initial voids ratios, low pre-consolidation loads, and very flat expansion curves, as shown in Fig. 2, which are consistent with the unusual physical characteristics shown in Fig. 1. An unloading and reloading cycle was made in the region just beyond the estimated pre-consolidation load in order to observe and study the characteristics of the consolidation curve in a loading cycle similar to that resulting from the sampling operation.

The marked effect of remolding or disturbance is shown by the fact that a compression of about 25% occurs under the pre-consolidation load as a result of complete remolding before equilibrium under this same pressure is again reached. The spread between the two curves is a measure of the natural structural capacity of the material. It is interesting to note in Fig. 2 that the breakdown of the natural structure and the dispersion of the soil grains by remolding, but without change in moisture content, also tend to decrease or slow up the rate of consolidation appreciably. The coefficient of consolidation is determined by the Taylor method,⁹ plotting compression against the square root of time. The parabolic nature of the initial stages of the time-consolidation curve of Fig. 2(b) is shown by the linear relationship to about 60% consolidation, which facilitates making preliminary forecasts of settlement.

The pressure-voids ratio curves of series 1 are plotted in Fig. 3 to show the range and characteristics of the materials of the deposits. A striking feature of these curves is the consistent tendency for steep curves of compressible materials to be associated with high values of the voids ratio at the 1 kg pressure, and for flat curves of relatively incompressible materials to be associated with low values. The practical significance of these interrelationships was first presented and discussed orally by Arthur Casagrande and Philip C. Rutledge, Assoc. Members, Am. Soc. C. E. (Fall Meeting of the Society in October, 1937), who suggested that a simple characteristic soil test such as the liquid limit could be used practically for obtaining preliminary estimates of the consolidation characteristics of undisturbed materials.

In a natural state of consolidation, a material passes from a suspension by the simple process of settling or sedimentation to a point where a real structure begins to form, which is capable of carrying stress as the overburden accumulates. A certain amount of pressure (although very small) is then required either to deform or to further compress the material. This moisture content is characteristic and may be considered as the starting point of true consolidation, and the transition from the suspended liquid state to the plastic state. The liquid limit, a characteristic moisture content of a remolded material, represents a moisture condition somewhat below this transition point. However, the same inherent physical characteristics of the material,

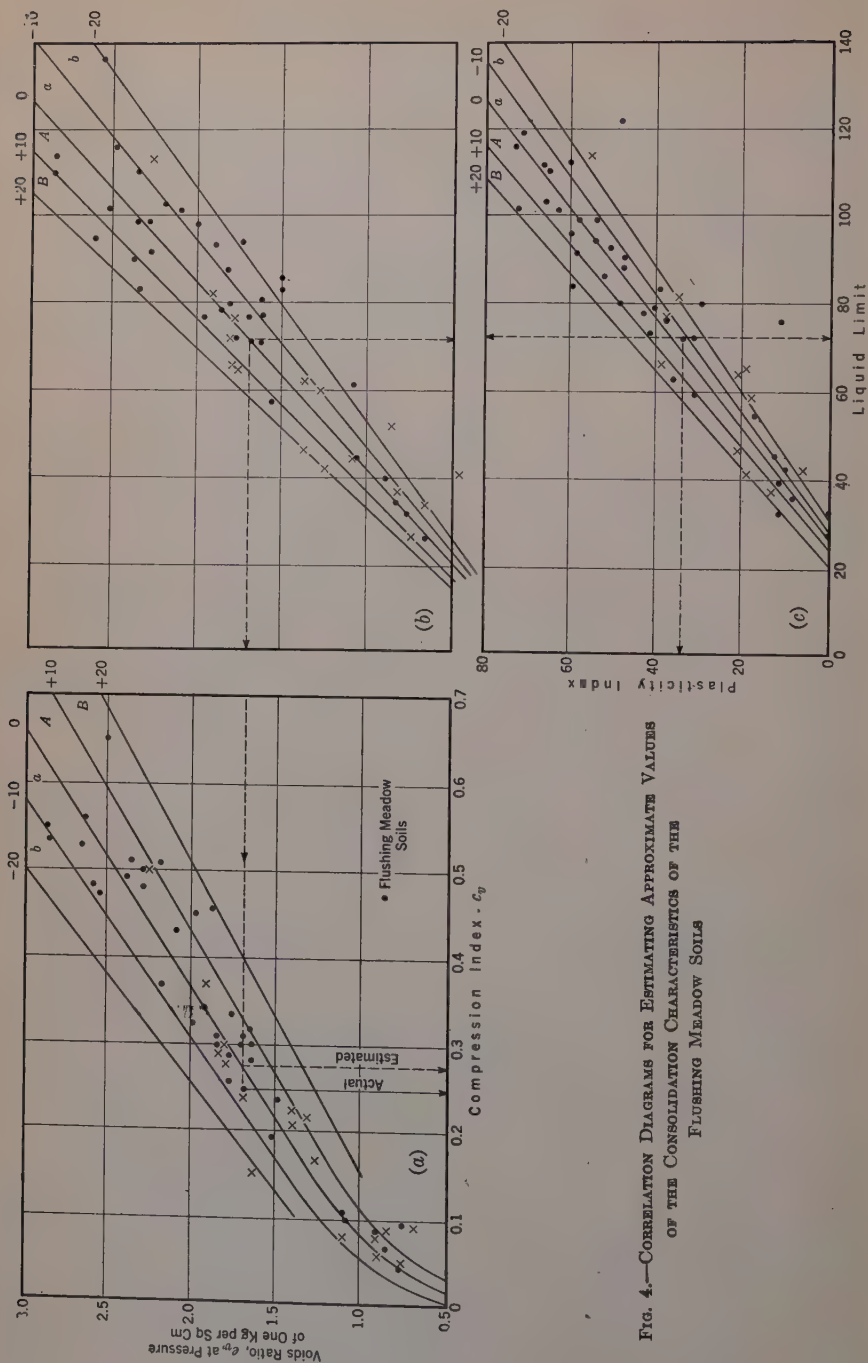


FIG. 4.—CORRELATION DIAGRAMS FOR ESTIMATING APPROXIMATE VALUES OF THE CONSOLIDATION CHARACTERISTICS OF THE FLUSHING MEADOW SOILS

which determine the liquid-limit condition, determine not only this characteristic moisture content at the starting point of consolidation before a real soil structure forms, but also the form of this structure, which is a determining factor in the compressibility of the undisturbed material in a natural state of consolidation.

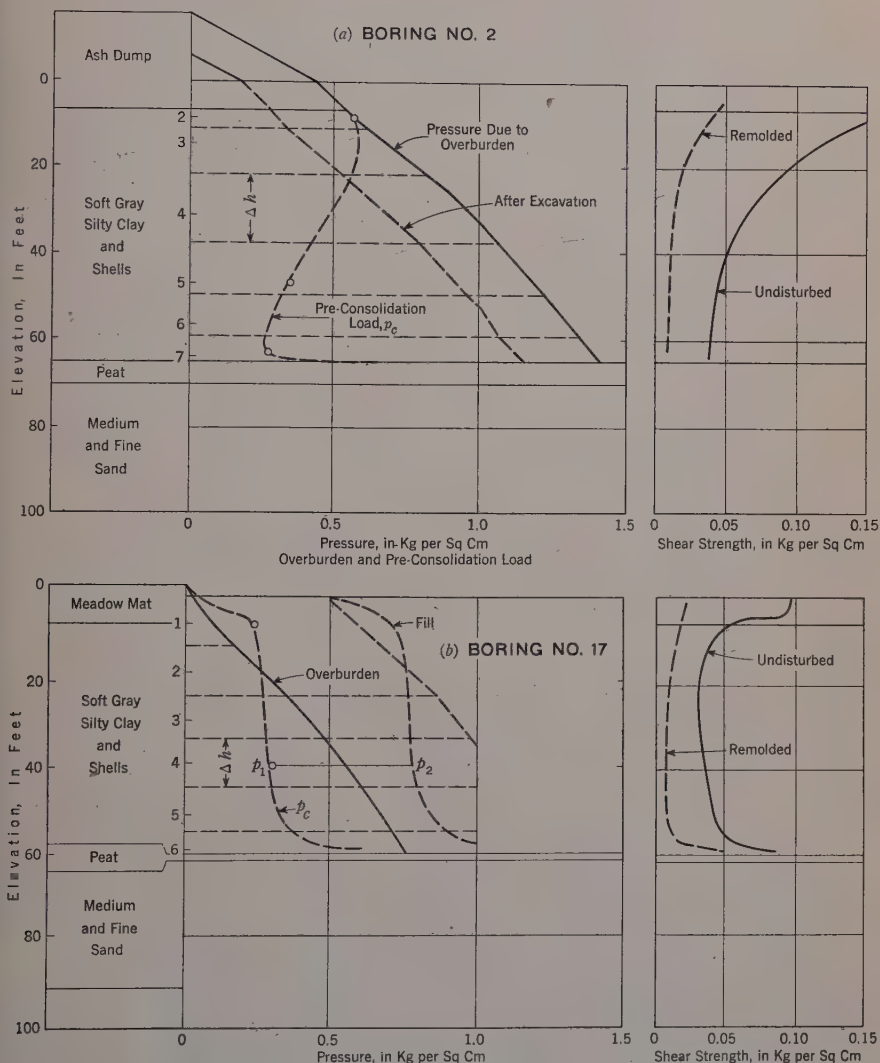


FIG. 5.—STRESS CONDITIONS AND SHEARING STRENGTH, FLUSHING MEADOW SOILS

A subsequent study of these interrelationships for the Flushing Meadow soils and for soils from other localities by C. R. Horne, Jr.,¹⁰ Jun. Am. Soc.

¹⁰ "Correlation of Compressibility and Atterberg Limits," by C. R. Horne, Jr., Thesis No. 494, Dept. of Civ. Eng., Columbia Univ., New York, N. Y., June, 1938.

C. E., is sufficiently interesting and of practical value to be included here in the diagrams of Fig. 4. About 60% of the values of the consolidation characteristics fall within a band of $\pm 10\%$.

Although the relations are not as consistent as desirable, studies seem to indicate that a material of low plasticity, falling in a zone (a) of Fig. 4(c), might be expected likewise to fall in corresponding parts of zone (a) of Figs. 4(b) and 4(a), tending as a consequence to have slightly lower values of both e_v and c_v . On the other hand, a material of more plastic nature than usual might be expected to lie in zone (A) of all diagrams of Fig. 4. These relations show the fundamental nature of the liquid and plastic limits as a characteristic moisture content in relation to consolidation phenomena, and indicate the need for fundamental research on the liquid and plastic limit tests to establish these relations more definitely for all types of materials. The effect of partial disturbance, inaccurate determinations of the liquid and plastic limits, and uncertainties associated with the consolidation test itself accounts for many of the discrepancies. An important fact to remember in all soil testing is variability of material which is always to be expected either in specimens taken from closely spaced samples in a boring or in different borings at the same elevation.

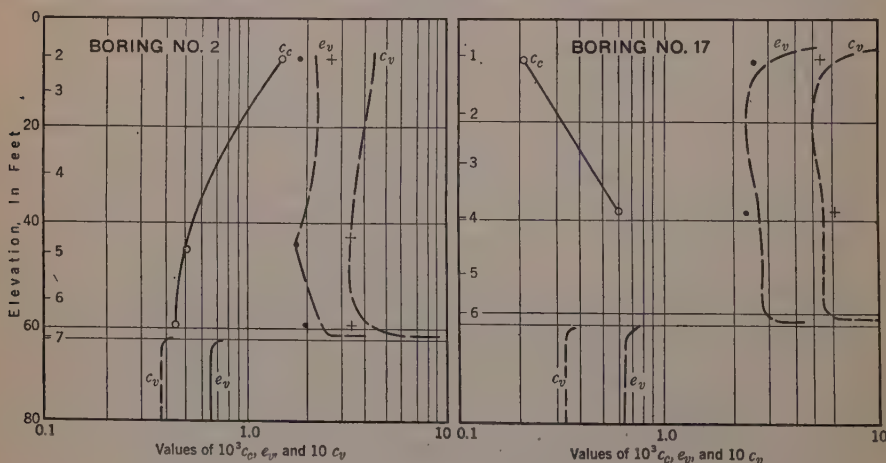


FIG. 6.—CONSOLIDATION CHARACTERISTICS, FLUSHING MEADOW SOILS

The possibility of making quite satisfactory preliminary estimates of the consolidation characteristics from the simpler routine soil tests, and of thereby extending and supplementing the information available for settlement analysis, is illustrated in Figs. 5 and 6. The profiles in Fig. 5 indicate the uniformity of the properties of the Flushing Meadow soils, the extremely compressible character of the meadow mat and the peat layer, and the relatively incompressible nature of the underlying silt and sand layers, the latter values being interpolated from other borings for comparison.

SHEARING CHARACTERISTICS

The major consideration in the development of the Flushing Meadow was to prevent and control subsurface movements in the soft material during

grading operations and in the construction of foundations, so as to preserve the natural structure of the material. The shearing characteristics of the Flushing Meadow deposits were obtained from results of direct shear tests of the "quick" and "delayed" shear types.^{11, 12, 13, 14} The first group of tests was

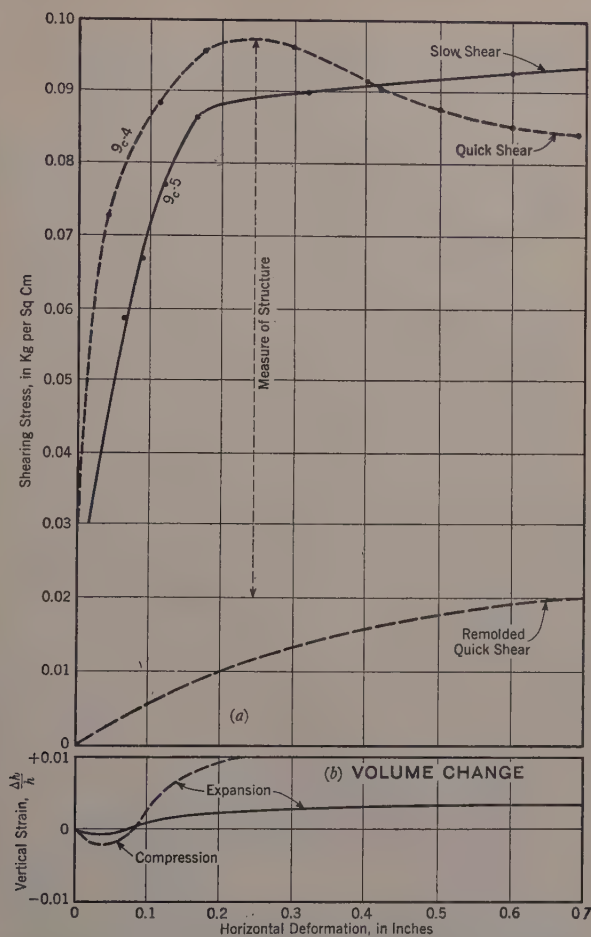


FIG. 7.—SHEARING STRESS-DEFORMATION AND VOLUME-CHANGE CURVES OF FLUSHING MEADOW SOILS AS DETERMINED BY DIRECT SHEAR TESTS OF THE STRAIN CONTROL TYPE

made in a ring type of shear apparatus with shearing load applied at a constant rate (water loading), and the later tests were made in a Harvard type of shear

¹¹"The Application of Theories of Elasticity and Plasticity to Foundation Problems," by Leo Jürgenson, *Journal*, Boston Soc. of Civ. Engrs., July, 1934, p. 206.

¹²"The Shearing Resistance of Saturated Soils and the Angle Between the Planes of Shear," by Kar Terzaghi, *Paper No. D-7, Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Harvard Univ., Cambridge, Mass., 1936, Vol. I, p. 54.

¹³"Die Coulombsche Gleichung für den Scherwiderstand biundiger Böden," von Karl Terzaghi, *Die Bautechnik*, 1938, Heft 26, p. 337.

¹⁴"The Shearing Resistance of Remolded Cohesive Soils," by M. J. Hvorslev, Assoc. M. Am. Soc. C. E., *Paper No. E-1, Proceedings*, Soils and Foundation Conference, U. S. Eng. Dept., Boston, Mass., June 17-21, 1938.

box with a strain-control loading at a rate of about 0.05 in. per min. The shearing stress for the undisturbed material in a rapid test in Fig. 7 increases characteristically to a maximum value and thereafter drops off to about 85% of the maximum. In the slow shear test (see Table 3), the stress increases

TABLE 3.—BASIC DATA FOR SHEAR TESTS IN FIG. 7

Sample No.	SHEARING RATE			Depth, in ft	MOISTURE CONTENT			Normal pressure, in kg per cm	SQUEEZE TEST	
	Description	Speed, in in. per min	Total time, in hr		Sample as received (<i>w</i>)	Liquid limit	Plastic limit		Undisturbed	Remolded
9 _c -4	Quick	0.05		42	116.5	100.8	56.7	0	3.0	4.6
9 _c -5	Slow		18 to 24	52	106.1	93.0	50.8	$\frac{1}{2}$	2.9	4.2

less rapidly with strain. It reaches about the same maximum value but shows only a slight dropping off. A ratio of maximum stresses in the undisturbed and remolded conditions of about 5 : 1 indicates a relatively undisturbed sample for these materials, and is a measure of their structural capacity.

The quick shear test values, together with the values estimated from the routine squeeze test, are given in a typical profile of Fig. 5. The shearing strength varies from about 0.03 to 0.08 kg per sq cm, and does not increase appreciably with depth for boring No. 17, from just below the meadow mat down to the sand and gravel layers. On the other hand, the shear strength of the upper layers in boring No. 2 is considerably greater, due to the consolidation of the upper layers under the weight of the ash dump; but at lower depths the shear strength is about the same as that found in other parts of the deposit. However, this weak material, strengthened near the surface by the meadow mat, was capable of supporting the initial blanketing (with a fill to a height of about 4 ft) and the grading equipment, without showing appreciable disturbance or mud-wave formation due to lateral subsurface movements.

To establish the relation between shearing strength and normal pressure for determining stable slopes, lateral earth pressure, etc., delayed¹¹ shear tests were made, specimens being pre-consolidated to a series of normal pressures and sheared under these pressures. Typical curves are shown in Fig. 8. The original series of delayed shear tests was made at a fairly rapid rate of 0.05 in. per min. The primary loading branch of the curve for sample 9_c-4 (depth 42 ft) passes through the origin, yielding a value of 20° 35' for the apparent angle of friction. Because insufficient time was allowed in the rapid test for the volume-change adjustments to take place during shearing, excess hydrostatic or neutral stresses^{12, 13, 14} were induced in the pore water, which could not be dissipated by the escape of water. This means that only part of the total applied normal stresses (possibly only 60% to 70%) were effective in mobilizing shearing strength.

Subsequently, slow shear tests were made on sample 9_c-5 from the same boring (depth 52 ft) at a very slow rate (about a 24-hr test), so that the hydrostatic stresses were at all times equal to zero. Therefore, the total applied stresses

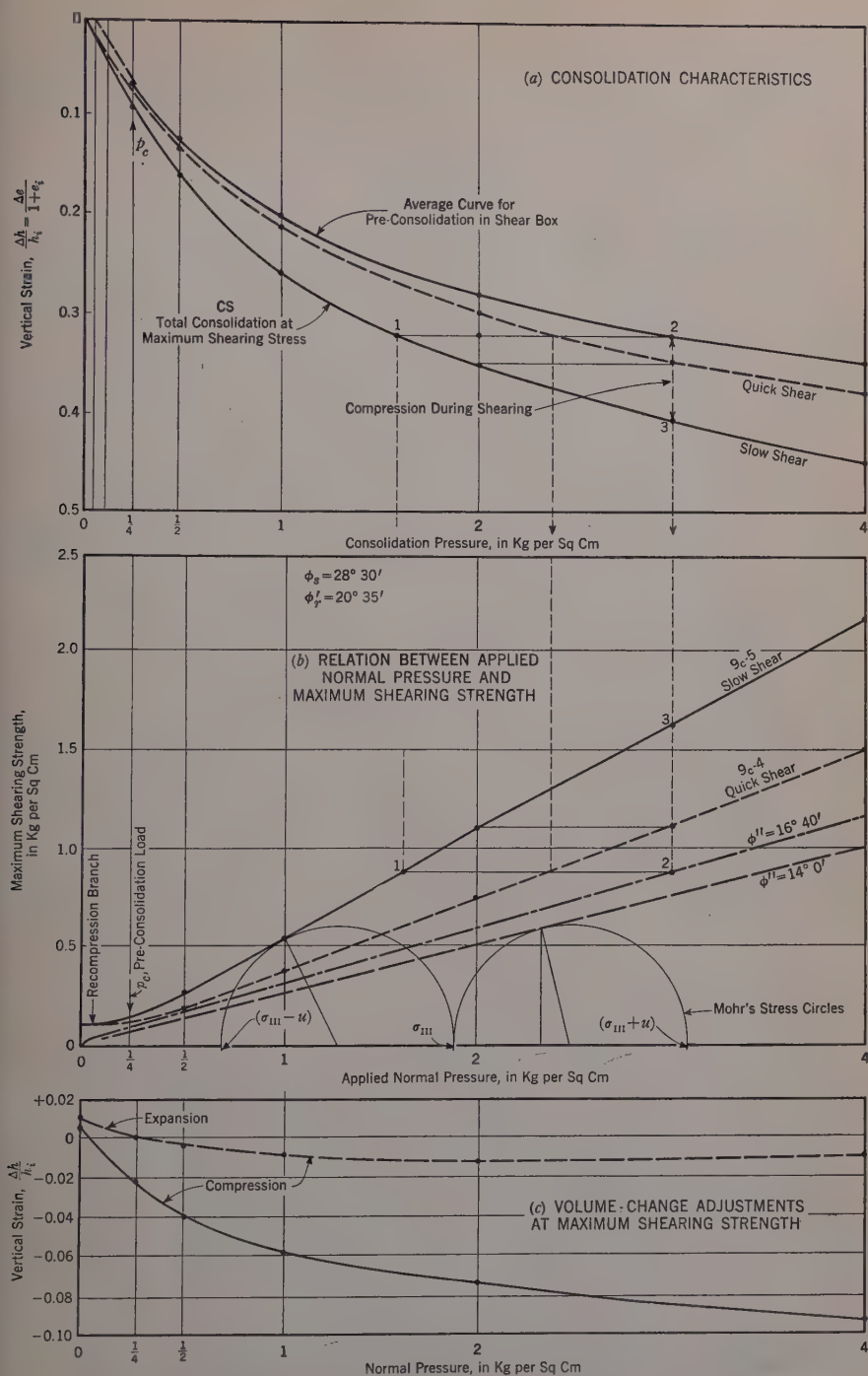


FIG. 8.—SHEARING CHARACTERISTICS OF THE FLUSHING MEADOW SILTY CLAYS AS DETERMINED BY DIRECT SHEAR TESTS OF THE STRAIN CONTROL TYPE

were effective stresses. The primary loading branch of the curve, expressing the relation between shearing strength and normal pressure, is a straight line that passes through the origin. This direct relation is typical for cohesive plastic soils in a natural state of consolidation, and Coulomb's simple law for cohesionless materials applies to the primary loading branch:

$$S = p_N \tan \phi \dots \dots \dots (3)$$

in which S is the maximum shearing strength, p_N is the effective normal pressure which equals the applied pressure in the slow shear test, and ϕ is the true angle of friction. The angle of friction obtained from the slow shear test on sample 9c-5 was $28^\circ 3'$. It is evident from the curve of volume-change adjustments in Fig. 8 that considerable consolidation takes place during a slow shear test. In a large mass of this material it is conceivable that practically no volume-change could occur during a rapid shearing failure. Under these conditions the apparent angle of friction, theoretically, may be half that of the slow shear test, or less. Also, due to the impermeable character of the material, the shearing strength would increase very slowly with consolidation and therefore the shearing strength is temporarily independent of any externally applied loadings.

Consolidation curve A represents complete consolidation under the applied normal pressures. The consolidation-shear curve CS , which is the sum of curve A and of the volume-change adjustment at maximum shear stress in curve B , represents the total consolidation required for equilibrium conditions at the maximum shearing stress.

In interpreting the results of the test, it must be remembered that the sample has been taken from its natural state with consequent relief of stress and expansion. Therefore, for normal pressures, less than the pre-consolidation load, one is operating on the reloading cycle of the consolidation and shear test curves. However, with the typical flat unloading and reloading branches of the consolidation curves of Fig. 2, the quick shear test gives a fair approximation of the shearing strength of the soil in the natural state, and the values are on the conservative side. Therefore, the quick shear test is useful for determining the shearing strength of the soil for such operations as the grading and filling of the Meadow.

Actual stress conditions approximating those in the natural state do not obtain until the normal pressure equals the pre-consolidation load. For greater normal pressures, the reloading curve then becomes tangent to, and coincides with, the primary loading curve, which passes through the origin. In analyzing a situation for controlling and preventing lateral subsurface movements in the deposit due to filling and grading operations, or due to construction of foundations, the profile of shearing strength in Fig. 5 is useful in locating the critical zone of maximum shearing stress as given by the stress conditions for different types of loadings of Leo Jürgenson¹¹ in relation to critical values of shear strength, particularly in view of the very low values just beneath the meadow mat.

SETTLEMENT ANALYSIS

It is necessary, in settlement computations, to make some estimate of the pre-consolidation load; that is, the probable state of stress in the soil in the

natural condition as shown in Fig. 2. Where three or more consolidation tests could be made per boring, a good idea could be obtained of this state of stress; but where fewer samples were tested, it is necessary to obtain an estimate of this stress condition for settlement computations. Because of the very flat expansion curve in consolidation and the fairly consistent relation between shear strength and pre-consolidation load, a correlation curve was obtained for these materials for estimating the approximate value of the pre-consolidation load from the results of the quick shear and routine squeeze tests. The curve was also very useful for estimating the place to make a loading cycle in the region of the pre-consolidation load.

The values of the pre-consolidation load determined from consolidation tests, and obtained from estimates, are plotted in the soil profiles of Fig. 5. It is interesting to note that below a depth of 10 to 20 ft the pre-consolidation load is approximately constant and considerably less than the weight of the overburden. There are two possibilities: Either the material is incompletely consolidated and stress is temporarily carried by hydrostatic pressure in the pore water; or there is permanent artesian water pressure in the sand and gravel layers extending up on either side to the flanks of the adjacent hills, a pressure gradient being established by upward flow through the silty clay deposit. These represent two limiting conditions for settlement analyses.

In the computation of settlement under a given external loading, it is convenient to divide the depth of the deposit into a number of increment sections of thickness Δh , as shown in Fig. 5, and take the appropriate values of pressure, p_1 and p_2 , at the center of each section from the pre-consolidation and external loading curves. The summation method can be made to take adequate account of the changes in the character of a deposit. Where consolidation tests have been made, the corresponding values of $(e_1 - e_2)$ are obtained directly from the pressure-voids ratio curves. Eq. 2 is then used directly in the settlement computations. Where the consolidation characteristics are estimated from Fig. 4, it is simpler to reconstruct the primary pressure-voids ratio curves, from the voids ratio at 1 kg of pressure and the voids ratio at 4 kg of pressure, which is equal to $e_v - c_v \log_e \frac{4}{1}$ or $e_v - 1.385 c_v$ (see Eq. 1).

CONCLUSIONS

The investigations have shown quite clearly that, if a limited amount of time is available for testing, it is very important to make a few carefully conducted consolidation and shear tests to obtain very complete information on the behavior characteristics of a few representative samples as control tests in the given area. The Flushing Meadow soils, although possessing unusual characteristics, exhibit physical properties consistent with their character. The simpler routine soil tests, carefully made on all soil samples, will not only give information on the nature and on the variability or uniformity in the character of the materials, but also will provide (after the correlation curves have been established for different soil properties by the control tests) a practical means for obtaining preliminary estimates on the consolidation characteristics and

shearing strength of all samples, and for extending and supplementing the information for settlement and stability analyses.

ACKNOWLEDGMENTS

The writer wishes to express his thanks and appreciation to Moran, Proctor, Freeman and Mueser, Consulting Foundation Engineers, for the opportunity of making the soil investigations in connection with the development of the Flushing Meadow Park site of the New York World's Fair, 1939. The testing was done in the Soils Laboratory of the Department of Civil Engineering at Columbia University, New York City.

CONCRETE IN SEA WATER: A REVISED VIEWPOINT NEEDED

BY HOMER M. HADLEY,¹ ASSOC. M. AM. SOC. C. E.

SYNOPSIS

Results of extended observations of concrete marine structures, along the Pacific Coast of the United States and Canada, are presented in this paper. No evidence is found of sea-water (sulfate of magnesium) attack on the concrete; where deterioration has occurred, it has been due to other causes. It is concluded from the uniform absence of sulfate attack in the wide range of concretes and cements involved in these structures that special precautions against sulfate attack are needless. It is concluded, however, that in sea-water concrete every care should be exercised to use sound materials, to obtain uniform density and impermeability of concrete, and to protect reinforcement from corrosion.

INTRODUCTION

The final criterion by which the behavior of anything in nature is to be judged is not the behavior of some similar thing; it is not the advance opinion of authority; and it is not the preliminary findings of laboratory investigation and research. The final criterion of behavior is no more or less than the actual behavior itself, and it is by their conformity and agreement with this actual behavior that the preliminary bases of judgment are to be appraised and assessed. If there is agreement, well and good. If there is disagreement, then it is the preliminary findings that must be adjusted and revised; it is not the actual behavior that must be denied.

This paper attempts to summarize the actual behavior of concrete marine structures along the Pacific Coast. How can that behavior be truly presented? When the mountain did not come to Mohammed, Mohammed went to the mountain. Nothing similar is possible here in the case of the average reader. The best that can be done is to submit representative views of structures taken in the very act of "behaving," in place, and to invite consideration of these

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by May 15, 1941.

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views in combination with other stated facts, all of which evidence, of course, is subject to quite independent check and examination by those to whom access to the structure in question may be available.

The writer is quite aware that there is a large body of opinion at variance with the views set forth in this paper. He regards it desirable, therefore, to focus attention on the question at issue—the behavior of concrete marine structures in service and the existence, in any form, of deterioration in such structures, apart from the enumerated kinds, which is definitely and peculiarly attributable to the sulfate action of sea water upon the hydrated cement of the concrete. Otherwise stated, the question is: Does any form of deterioration occur in the concrete in sea water that would not develop if the exposure were changed to similarly agitated, similarly fluctuating, fresh water? In the considerable body of evidence that exists along the Pacific Coast, the writer is aware, with noted exceptions, of none whatever. It is also to be observed that it is with sea water in its normal range of concentrations—not with other sulfate waters—that the paper is concerned.

THE PREVAILING VIEWPOINT

"Many engineers who are most experienced in marine construction are of the opinion that all concrete structures in salt water, and a considerable proportion of those near salt water in warm climates, deteriorate steadily and surely."²

These words express a widely accepted opinion that deterioration is inevitable. It is supposed to result from the combination of sulfates of magnesium in the sea water with free lime in the hydrated cement and from the consequent formation of calcium sulfate which either by dissolution or expansion disrupts the concrete and effects its destruction. Periodically, in the technical press, reference is made to the sulfates of magnesium in sea-water attack on concrete in the most casual, matter-of-fact manner, as if it were definitely proved—an established reality. Remedies are proposed for it.

Under these conditions it is, to say the least, extraordinary that nowhere along the Pacific Coast, from Canada to Mexico, has the writer been able to find any evidence in concrete marine structures that an attack of such a nature occurs. Deterioration of various kinds assuredly is to be found. There is much of it in certain localities. Sometimes it is very serious; sometimes it is minor and trivial. None of it, however, is of a character to satisfy the sulfate of magnesium-free lime explanation which Messrs. Atwood and Johnson state³ was advanced by L. J. Vicat. They quote E. Candlot as follows:

"According to Vicat, it is the action of this salt [sulfate of magnesia] to which must be attributed the rapid decomposition of lime and of certain cements by sea water; the sulphate of magnesia combines with the free lime of the mortar and is transformed into sulphate of lime; the magnesium is precipitated. If the current of the water which traverses the mortar be somewhat rapid, the sulphate of lime is carried out, a new quantity of lime enters into solution, is in turn transformed into sulphate, and the *gange*

² "The Disintegration of Cement in Sea Water," by William G. Atwood, M. Am. Soc. C. E., and Arthur A. Johnson, *Transactions, Am. Soc. C. E.*, Vol. LXXXVII (1924), p. 205.

³ *Loc. cit.*, p. 207.

thus decomposes continuously and finishes by forming only a sandy mass. When the current is weaker or is produced only at rather long intervals, the lime sulphate may crystallize and thus cause the expansion of the mortar. It is to this cause that we may attribute the phenomenon of expansion of mortars decomposed by sea water."

From the foregoing it appears that there should be either a progressive leaching out of the calcium sulfate with a correspondingly developed porosity and weakening of the concrete, or the concrete should become swollen, disrupted, and disintegrated by crystallization of the calcium sulfate. Furthermore, although there is no definite statement of the time rate at which these changes occur, it is to be assumed that they take place at a rate sufficiently rapid to show some indication of this deterioration after a reasonable period of time—say twelve or fifteen years. Certainly one should not be compelled to wait longer than that to find the first traces of it if it is a matter of any practical consequence.

With the understanding, then, that the characteristic form and manifestation of sulfate of magnesium attack on concrete is either a developed porosity or a swollen disruption of the mass and, after extensive inspections and examinations of many structures which uniformly fail to reveal deterioration of the requisite type, it is repeated that there is no evidence in concrete exposed to sea water along the Pacific Coast to substantiate the sulfate of magnesium-free lime theory. The confirmation is not to be found, however diligent the search. On the other hand, when a large amount of concrete (made with many different brands of portland cement) is seen that has been subjected to full sea-water action for fifteen, twenty, or more years and still has the original wood grain and form marks showing in its surfaces; still has clean, sharp corners and offers flinty, hard resistance to hammer blows; and is utterly lacking in deterioration, there is no avoiding the conclusion that the sulfate of magnesium-free lime deterioration theory is deficient in that indispensable necessity of all valid theories—proof. If, as is said, sea water attacks concrete, why does not all concrete deteriorate? Why, even in deteriorated structures, is most of the concrete free from deterioration in spite of having the same identical exposures as the deteriorated parts?

CONCRETE A GENERIC TERM

It would be well at the outset to consider what concrete is and to recognize that the term is a broad and generic one. Concrete is the name given to any hardened mixture of portland cement, water, and fine and coarse aggregate. If the coarse aggregate is lacking, the mixture is called mortar, but otherwise the name concrete is given, regardless of wide variations in the proportions of the several components of the mixture and regardless of the manner and method by which it is mixed and placed in its forms. Whatever it is, it is called concrete as long as its material components are as stated. The consequence of this convention is that the behavior of concrete under any particular set of conditions may vary widely, depending upon the particular concrete that is subjected to the exposure. Quite similar would be the variable behavior of rock which, if it were hard granite, would behave in a markedly different manner than rock which was a soft sandstone.

It is to be expected that a wide range of mixes has been used in concrete marine work, and such indeed is the case. Likewise variable have been the methods of placement. It is apparent in many structures that care has been exercised to place the concrete uniformly and without segregation. It is apparent in other structures that little or no concern was felt for the attainment of these objectives: The concrete was dumped into the forms in a carefree, happy-go-lucky fashion, contentment abiding in the knowledge that "sufficient unto the day is the evil thereof." In reinforced structures the thickness of the protective covering over the bars varies in different jobs as does the quality of concrete in the covering. Out of all these variables, manifestly, there are opportunities for diverse behavior of "concrete in sea water," and it is not surprising that deterioration of various kinds has developed.

THE SEA-WATER EXPOSURE

The sea-water exposure itself varies widely. In the most sheltered and protected locations there is the rise and fall of tides with alternate wetting and drying of the concrete. Waves, swells, and the wash of ships frequently amplify the tidal wetting. Flotsam and drift are tossed about by the water and deliver mechanical blows. On the open ocean coast, storm waves themselves deliver tremendous thrusts and blows against the structure and may roll and grind loose boulders against it; moreover, along the Pacific Coast, from San Francisco (Calif.) north, the forest mantle of the country yields great quantities of drift which sometimes may be whole logs and trees that form heavy battering material. Additional to this along some coasts are the disruptive effects of ice and freezing. All in all it must be recognized that, quite as a marine exposure is a poor place for rock of a soft shale, indurated mud type, so, likewise, is it unsuited for lean, porous concrete of segregated, random structure.

DETERIORATION BY MECHANICAL ATTACK

Indeed the most striking and characteristic forms of deterioration to be found in concrete exposed to sea water are those due to deficiencies in the quality and structure of the concrete itself: Weak concrete is abraded, honey-combed areas become full cavities, laitance seams become extended voids, construction joints and pour planes tend to chip and widen at their edges, and sharp corners and projections become broken and knocked off. The processes of attrition and erosion by which, in the long reaches of time, all things are reduced to fragments and to dust are in aggressive action on the ocean's shores, and that which is vulnerable to attack is speedily revealed. On the other hand, strong, sound concrete that is uniformly dense and impermeable shows but slight change in a quarter century or more and quite merits the term "permanent" in the time scale of human affairs.

Such contrasting aspects of concrete in sea water obviously result from variations in the concrete itself. They have nothing to do with sulfates of magnesium in the sea water, nor with the sulfate-resistant qualities of cement. Deterioration such as that described is due to physical and mechanical causes and could, and would, occur as well if the ocean were of fresh water instead of salt water. Unfortunately, a great deal of writing has indiscriminately

lumped all forms of deterioration together as "sea-water attack," whence the conclusion is readily drawn that it is the sea water itself that exercises a peculiarly malign effect upon the concrete. For resistance to this, the commonest form of deterioration in concrete exposed to sea water, it is safe to say that making concrete which is fresh-water resistant or frost resistant elsewhere will furnish full and adequate protection.

RUSTING OF REINFORCEMENT

In reinforced structures, the second most common form of deterioration is that due to rusting of reinforcement, disruption of the covering concrete, further corrosion of steel with loss of section, loss of bond, etc. It develops most frequently in members of small section—piles and the stems of beams and girders. Unprotected steel rusts. It rusts very slowly in a dry climate and more rapidly in a moist climate. It rusts most rapidly and heavily of all along the sea coasts where the air may carry minute drops of salt water in suspension, caught up from waves and surf. In such an atmosphere, rust will form quickly on reinforcing bars wherever the covering concrete is porous or absent. If possible it will develop progressively along the bar and with its inevitable expansion will split and disrupt concrete. The primary remedy is to provide dense, impermeable concrete and adequate cover. Around salt water is a poor place for scant cover over reinforcement, for misplaced reinforcement, or for lean concrete. However, it is to be noted that, although the rusting of reinforcement is indeed aggravated and accelerated by the salt-water exposure, it is wholly independent of the sulfate-resisting properties of the cement used in the concrete. Consequently, here again, primary importance attaches to density and impermeability and to the thickness of the protective concrete rather than to other things. Additional protective measures that may be accepted as thoroughly effective are the asphaltic impregnation of concrete piles, extensively used by the Los Angeles (Calif.) Harbor Board,⁴ and the ingenious system of applying an asphaltic coating to the underside of a concrete deck, developed and used by the engineers of the San Francisco Harbor Board.⁵ To the best of the writer's knowledge, these methods have been 100% effective in accomplishing their respective objectives.

SOLVENT ACTION

Occasionally there is evidence of a slight solvent action of sea water on concrete. The original surface film of cement has disappeared, or a honeycombed spot weakens and loosens some pieces of aggregate. In time, porous concrete may appear to become increasingly porous. Such changes are not peculiar to sea-water exposure, however. They can occur equally in fresh water. Long-time tests made in the testing laboratory of the City of Seattle, Wash., about 1925, by T. H. Carver, M. Am. Soc. C. E., clearly showed a minute but definite dissolution of concrete in flowing fresh water, which dissolution did not occur in still water. In Fig. 1(a), the cylinders on the left were stored in a tank through which a slow steady flow of Cedar River water was maintained. Those on the right (Fig. 1(b)) were stored in still water in sealed jars. The mixes were, left

⁴ *Civil Engineering*, November, 1931, p. 1245.

⁵ *Engineering News-Record*, June 8, 1939, p. 771.

to right, 1-1-2, 1-2-4, and 1-3-6, respectively, in both Fig. 1(a) and Fig. 1(b). After five years the standard-cured 2-in. by 4-in. cylinders, in Fig. 1(a), all showed evidence of attack. The specimens of 1-3-6 mix had entirely lost their surface films of cement paste; the 1-1-2 specimens could have theirs



FIG. 1.—WIRE-BRUSHED, 2-IN. BY 4-IN. CYLINDERS AFTER FIVE YEARS IMMERSION IN FRESH WATER

removed easily by wire brushing. The companion specimens, in Fig. 1(b), stored in still water in sealed jars, were unaffected. The loss of strength, running-water storage compared with still-water storage, was markedly greater with the lean mixes than was the case with the richer mixes. The desirability of an impermeable concrete was clearly shown. In the case of sea water, there is a continual change of water with tides, currents, surf, etc. There is ample evidence from both laboratory and field investigations of the need for density and impermeability in concrete exposed to sea water, and also in concrete exposed to fresh water.

FREEZING

Disruption of porous concrete by the freezing of absorbed water does not occur along the Pacific Coast of the United States, not because there is no porous concrete to be found, but because freezing seldom, if ever, occurs. Such disruption of porous concrete by freezing is quite independent of sea water, however. It occurs with both fresh and salt water.

EVAPORATION

Disruption may likewise develop with porous concrete in the tidal zone or above if evaporation produces seriously harmful concentrations of sea water in the pores. The late C. S. Pope,⁶ M. Am. Soc. C. E., has presented a view of the top of a sea wall that shows scaling and deterioration which were undoubtedly due to concentrated saline solutions and crystallization produced by evaporation. This is a form of deterioration which is indeed peculiar to sea water and not to fresh water. In general, however, it is of limited occurrence, since the usual freedom of drainage appears to prevent it. The flat top of the

⁶ *Transactions, Am. Soc. C. E.*, Vol. 98 (1933), p. 511.

sea wall cited by Mr. Pope acted to trap and hold a certain amount of spray thrown upon it, and with rather porous concrete and aggregate the consequences were as shown in Fig. 1.

UN SOUND MATERIALS

Still another type of deterioration—that due to the use of unsound materials in the manufacture of concrete—is to be found. This is characterized by a pronounced general expansion of the entire body of the concrete and much cracking. There are two distinct varieties of this classification. The common one is that in which the concrete remains hard and is entirely unaffected by the sea water. In the other variety the concrete softens into a mushy mass practically devoid of cohesion.

Concrete is composed of cement, water, fine aggregate, and coarse aggregate. The coarse aggregate is a highly unlikely cause of a general and all-pervasive expansion in a concrete structure free from local poppings and pittings and may be dismissed from consideration in all but exceptional cases. So, likewise, are differences in mixing waters, although, as the agent initiating the hydration of cement, mixing water may be regarded as a cause. Perhaps strongly alkaline waters under some conditions may be unfit for use, although sea water itself, notably in the case of the Florida East Coast Railway Extension to Key West, has been used as mixing water in a number of instances with no appreciable ill effects. There remain the cement and fine aggregate. The latter is almost always a natural sand produced by weathering and erosion of a great variety of rocks, and is transported, intermingled, and deposited by streams. Depending upon their origin, sands from the same pit may be of definitely uniform or of utterly diverse and random mineralogical characteristics. It is a common and usually valid assumption that natural sands are stable, sound, and chemically inert, but this assumption, easy though it is to make, is not necessarily true. It is cement and sand, individually or in combination, that must be considered as possible causes of the expansion referred to.

Improperly manufactured cement may indeed be the cause of the trouble. Nevertheless, the adoption of modern improvements in manufacturing and testing procedure should give the engineer increased confidence in the capability of portland cement to meet fully all reasonable construction requirements. More careful grinding and particularly blending of the raw materials insure more uniform and correct burning and therefore a more dependable product. New and more severe tests for soundness detect any tendency toward delayed unsoundness. However, the cases of deterioration in existing marine structures in which serious expansion has occurred are not numerous, and those in which this expansion is properly to be charged to the cement alone—that is, wherein the aggregate is not partly or completely responsible—have yet to come to the writer's attention.

Expansion of the type in which the structure cracks but the concrete remains hard and not observably affected by sea water is discussed and illustrated in a series of valuable papers by Thomas E. Stanton,⁷ M. Am. Soc. C. E. The original paper, presented at the annual meeting of the Society in January, 1940, describes a typical case of such expansion in a sea wall in Ventura County,

⁷ "Influence of Cement and Aggregate on Concrete Expansion," by Thomas E. Stanton, *Engineering-News Record*, February 1, 1940, Fig. 2, pp. 59-61.

California. In March, 1940, the writer discussed^a the condition of this sea wall and others nearby. There appears to be no reasonable doubt that the nature and cause of deterioration of this kind have been definitely established by the notable work of Mr. Stanton and his associates. This form of expansion has developed in bridge piers and abutments in fresh-water streams—for example, the highway bridge across the Salinas River at King City, Calif.

A. A. M. Russell, Assoc. M. Soc. C. E., has made some tests which brought into definite and almost dramatic focus hitherto confused and obscure causes of trouble of the expansion-softening type. From a pit from which had come the sand used in a waterfront structure that developed very serious deterioration of this type in a few years' time, he obtained sand samples, and for comparison he used sand from another pit, several hundred miles distant from the first pit, and of known soundness. Petrographic examination of the questioned sand revealed a heavy percentage (more than 40%) of feldspar grains which proved to be of a highly unstable nature.

As an accelerated test, equal samples of both sands were put in beakers of sea water, which were then placed on an electric plate and held continuously at a temperature slightly below boiling. In four days about half of the questioned sand had broken down and changed completely to a mud, whereas the other sand sample showed only a thin film of mud across its top surface. Then, as a direct sea-water test, briquet specimens were made up of the questioned sand and of standard Ottawa sand. The latter, consisting entirely of particles passing No. 20 and retained on No. 30 sieves, suggested a similar grading for the other sand. Consequently, it was sieved into three gradings and three different mixes were prepared. In one mixture only particles passing the No. 10 and retained on No. 20 sieves were used; the second had only particles passing No. 20 and retained on No. 30 sieves—that is, the Ottawa sand grading; the third mixture had a grading with particles ranging uniformly from fine to coarse. After tension tests were made, the briquet fragments were placed in jars of sea water and were kept at room temperature, the sea water being renewed occasionally as evaporation took place. In less than a year the Ottawa sand specimens were wholly unaffected, having hard sharp edges and no trace of deterioration. The same was true of the specimens of the questioned sand that had the fine-to-coarse grading and were therefore of reasonably dense structure. The specimens made with the No. 10 to No. 20 mesh particles were beginning to soften slightly on their edges but were in fair shape. The No. 20 to No. 30 mesh samples, on the other hand, were in an advanced state of disintegration. They had swelled, cracked, were slimy on their surfaces, and their edges were soft and crumbly. These specimens, being directly comparable in every respect to the specimens made with Ottawa sand and with the same cement from the same sack (which Ottawa sand specimens were wholly unaffected), showed conclusively, in conjunction with the preceding accelerated test on the sand, that the unsoundness in this case was in the sand and not in the cement.

Inasmuch as the deterioration produced closely resembled that in the structure built with this sand, it appears to be established, at least in this case, that

^a *Engineering-News Record*, March 14, 1940, p. 45.

the sea-water attack was on the sand and not on the cement. It is scarcely conceivable that this trouble could have been avoided by special sulfate resistance or other special qualities in the cement. It is a fact that serious deterioration of this type appears to have developed in a section of the coast of rather marked geographical limits. It should be added that it is only in sea water that the writer knows of the occurrence of this form of deterioration, and indeed it may be that this feldspathic sand is particularly unstable in sea water. Its use for any concrete anywhere, however, appears to be questionable.

There are a few cases known to the writer where concrete in exposed locations appears to have developed expansion troubles somewhat intermediate between the two types—that is, there has been cracking of the type described in Mr. Stanton's papers with apparently a certain softening of the concrete. This has resulted in the complete destruction of parts of the structures. Extensive cracking of exterior surfaces due to expansion of the interior of the mass, of course, greatly weakens resistance to flexural stresses; similarly, such complete prevention of longitudinal expansion in certain sections as to lead to crushing at the ends would practically destroy their resistance to the pounding of waves at such points. Whatever the exact cause of the trouble in these cases, there is no uncertainty about abnormal expansion; it is to be avoided and eliminated. It is believed that the preceding covers all the main types of deterioration that are to be found in Pacific Coast concrete marine structures.

SUBJECTIVE DETERIORATION

Another trouble of concrete in sea water has nothing to do with the sea water. This trouble arises from hasty observation and incomplete reporting of conditions. In glowing zeal to establish either broad generalizations or the urgent need for the adoption of certain safeguards, only as much of the evidence is adduced as supports the contention. This frequently creates in readers' minds an entirely erroneous idea of conditions.

Fig. 2 shows a view of an old quay wall at the Puget Sound Navy Yard, Bremerton, Wash., built in 1901 and known as quay wall D. It extends from the stepped-off joint at the reentrant angle to the corner at the extreme left. At the right of this section and in the right foreground of the view is a portion of quay wall E, constructed in 1896. The deep cavities and extremely bad conditions at the base of wall D are clearly apparent. Likewise clearly apparent is the condition of the wall above the base portion which is extremely hard and today (1940) shows no trace of deterioration, the wood grain and form marks being discernible through the coating of fuel oil which now covers it. An official statement regarding this condition is the following:

"The bottom part of the west wing wall from the stepped off joint to the south corner was poured in 1901 with a tremie up to a point where the water could be expelled from the open cofferdam, probably three feet higher than the water level in the photograph."

Regarding this same wall, the following is the complete report,⁹ published in 1924 by the National Research Council:

"Quay Wall D—This is a gravity type wall built in 1901-02. Local aggregates and condon [Condor] cement were used in the proportions of

⁹ "Marine Piling Investigations," National Research Council, 1924, p. 403.

1 : $2\frac{1}{2}$: 5. This concrete was hand mixed and deposited below water level by tremie and above with wheel barrows.

"The inspection of Mar. 8, 1923, showed this wall to be badly deteriorated, though 'Gunitite' repairs had been made."



FIG. 2.—QUAY WALL D, PUGET SOUND NAVY YARD, BREMERTON, WASH.

Although this report is literally true, it nevertheless creates a wholly erroneous impression since no reader would know that the upper part of the wall, also in sea water, made with the same cement as the base, was practically free from deterioration on March 8, 1923. With the same construction methods, any and every cement regardless of type or composition that might have been used in this wall would have been similarly attacked or not attacked by the sulfates of magnesium in the sea water, depending on whether it was dumped through a tremie tube or deposited by wheel barrows. Many reports of sulfate-of-magnesium attack on concrete in sea water are of the foregoing character.

SUMMARY

The harm resulting from misunderstanding a natural phenomenon arises from the likelihood that measures adopted to effect changes and alterations in it will be misdirected. There is no particular reason for research and for grave alarm over the strong links of a chain so long as there are weak links that need strengthening. Not until the real hazards of sea-water use are definitely foreseen and are guarded against is there need for such extreme refinements as protecting against sulfates of magnesium—and no particular harm will result if this safeguarding is completely neglected.



FIG. 3.—TYPICAL VIEW OF SUBSTRUCTURE OF FERRY PIER, NORTH VANCOUVER, B. C., BUILT IN 1909

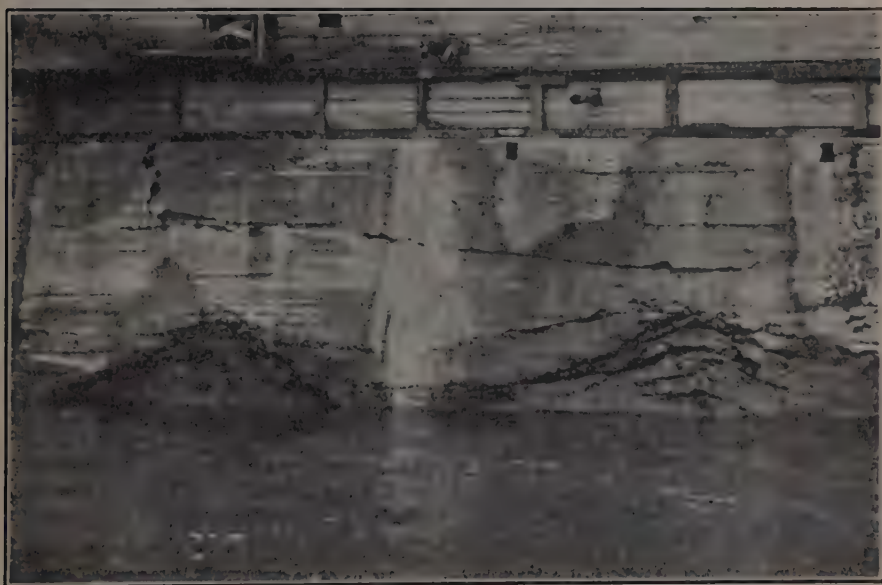


FIG. 4.—DETAIL OF OLD WATERFRONT WALL, SEATTLE, WASH., BUILT IN 1913

This paper is based upon visual observations of concrete structures in sea water along the Pacific Coast and is concerned with sea water only. It is intended that its statements and conclusions apply to sea water only—not to those sulfate waters and exposures elsewhere which assuredly require special measures and treatments. The dearth of supporting laboratory data may be regarded as somewhat unusual, but it is hoped that the references to the work of Messrs. Carver, Stanton, and Russell will supply this deficiency in some



FIG. 5.—BROADWAY PIER, SAN DIEGO, CALIF., BUILT IN 1912

measure. After all, it really is not necessary to have laboratory confirmation of that which is spread obviously before one's eyes.

Probably no better ending could be made to this paper than to append a short list of some representative older concrete-in-sea-water structures, variously located, which exemplify the behavior of sound, impermeable concrete and which may be examined by any one. Several views are shown in Figs. 3 to 7. These structures are not without flaw or blemish. Some of them have bad spots such as patches of honeycomb dating from original construction, or have been worn by abrasion. However, they have had twenty or more years of sea-water exposure, and their concrete is substantially free from trace of sea-water attack. They were built at different times, with different brands of



FIG. 6.—PIER ON SAN FRANCISCO WATERFRONT, BUILT IN 1909



FIG. 7.—DETAIL OF ABUTMENT OF EAST NEAPOLITAN BRIDGE IN NAPLES DISTRICT, LONG BEACH, CALIF., OFF ALAMITOS BAY, ERECTED IN 1913

cement, which assuredly varied in their percentages of tricalcium aluminate and other compounds, and with different construction methods. After the examination let the observer ask himself the question: Where is the sulfate-of-magnesium attack? Assuming that there is only one conclusion—namely, that he has not seen any—it is then fair to ask why it is not to be found. It is to be hoped that he will look for it elsewhere—not in books, but along the waterfront. This paper is intended to present the idea, definitely, that such an attack on concrete in sea water does not occur and therefore need not be guarded against. On the other hand, it would re-emphasize what many observers have stated before—the need of adequately protecting reinforcement from corrosion and of having dense, impermeable concrete made of sound and enduring materials.

Representative older structures evidencing no sulfate-of-magnesium attack upon their concrete are the following:

Location:	Built:
North Vancouver, B. C., Canada, Ferry Pier.....	1909
Victoria, B. C., Canada, Dallas Road sea wall.....	1911
Seattle Wash., sea wall south of Madison Street.....	1916
Bremerton, Wash. (Puget Sound Navy Yard) {Quay wall E..	1896
	Quay wall D.. 1902
San Francisco, Calif. {Cylinder substructure, pier 38.....	1909
	Pre-cast pile substructure, pier 17..... 1912
Los Angeles, Calif. {Lighthouse base, San Pedro breakwater..	1910
	Piers of Southern Pacific Railway bascule
	bridge at West Basin..... 1912
San Diego, Calif., Broadway Pier (see Fig. 5).....	1912

Reports of 27-yr tests of concrete specimens, made with five different brands of cement immersed in sea water on the San Pedro breakwater at Los Angeles,¹⁰ were published in 1932. These specimens were made in 1905 by the U. S. Engineer Department and were exposed in the tidal zone. No evidence of sulfate-of-magnesium attack was found in them nor any deterioration other than abrasion due to mechanical causes.

Fig. 4 is a view of an old sea wall in Seattle, Wash., built in 1913. Part of the wall shows what is commonly called "sea-water attack," whereas most of the wall shows no attack. The cause of the trouble is self-evident.

In Fig. 6(a), honeycombed concrete and sound concrete are shown side by side in the same cylinder, and both conditions occur at the water level. Fig. 6(b) shows another cylinder that is honeycombed at the high-tide line and also above that line.

CONCLUSIONS

Because of complete lack of evidence in concrete marine structures of sulfate-of-magnesium attack upon them, there appears to be no valid basis for believing that such attack occurs.

The vitally important matter for concrete used in marine work is to have dense, impermeable concrete made of sound materials with adequate cover over reinforcement.

¹⁰ *Western Construction News*, June 25, 1932, p. 367.

DYNAMIC STRESS ANALYSIS OF
RAILWAY BRIDGESBY R. K. BERNHARD,¹ M. AM. SOC. C. E.

SYNOPSIS

Since it is a known fact that stresses due to dynamic impact often become an important factor in the design of a railway bridge, it is the purpose of this paper to indicate a simple method by which these stresses may be determined. In order that this method may be presented in the shortest manner, it will be restricted first to the main trusses or girders of single-track spans with known static loads. It is assumed that, as a steam locomotive crosses such a span at the "critical speed," the greatest vibrations are set up by the action of the free and unbalanced centrifugal forces of the moving parts. The various formulas deduced herein are set up in the form of nomographic charts. Seven of these are located side by side for ease of manipulation. If the damping capacity of any vibrating structure is known, the true value of the stresses due to impact can be read from the chart. One problem is worked through, showing the advantages, and also the limitations, of the method. Finally, a comparison is given with the impact percentage of standard specifications.

INTRODUCTION

One of the members of a steel railway bridge in Europe failed recently while a train of comparatively light weight was passing over it at a relatively low speed. The result has been a revival of the discussion of dynamic overloads. This question is often "sidetracked," particularly by practical field engineers, since purely mathematical methods²⁻¹⁰ of developing the analysis may become extremely involved.

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by May 15, 1941.

¹ Prof. and Head, Dept. of Eng. Mechanics, Pennsylvania State Coll., State College, Pa.

² "Discussion of Differential Equations Relating to the Failure of Railroad Bridges," by G. G. Stokes, *Transactions*, Cambridge Philosophical Soc., Vol. VIII (1849), p. 707.

³ "Vibration of a Girder Under Moving Loads," by H. Zimmerman, W. Ernst and Sohn, Berlin, 1896.

⁴ "Forced Vibrations of Prismatic Bars," by S. Timoshenko, *Journal für Mathematik und Physik*, Vol. 59 (1911), p. 163.

⁵ "Impact in Steel Railway Bridges of Simple Span," by J. B. Hunley, A.R.E.A., Vol. 37, No. 380, October, 1935.

(footnotes continued on next page)

Most impact formulas of standard specifications, which make an allowance for the dynamic increment in the design of bridges, are derived more or less by experiment. Experimental methods for finding the impact value by field tests, using various types of measuring apparatus, are usually rather time-consuming. Even if the matter is approached from a strictly mathematical point of view, only approximate solutions can be obtained. For some time past many investigators have attempted to find a method of computing this impact value which would combine sufficient accuracy with ease of manipulation. One of the most comprehensive papers on this subject was published by J. B. Hunley in 1935.⁵

Following are nomographic charts which have been developed leading to an approximate solution of the dynamic problem by combining theoretical investigations with the results of numerous field tests that have been conducted both in the United States and in Europe.

There are various methods of including the impact factor in the design, but the simplest seems to be to make the computation in two steps: First, the structure should be designed for dead load and moving load in the usual static manner; and second, a supplementary computation must be made of the stresses caused by those locomotives which produce the maximum dynamic effect. This second computation is often the determining factor because, although the static investigation is based on the heaviest moving loads, larger dynamic stresses due to resonance effects may be set up by light or intermediate loads.

The computation as given herein does not provide a general solution for all possible cases. For the sake of simplicity, the method is made applicable only to such conditions as often occur in designs. Among these may be mentioned single-track, simple spans from 60 to 300 ft in length.

ASSUMPTIONS

The following assumptions are made:

(1) For main bridge girders the largest dynamic effect will occur at the critical speed. This critical speed is that at which the period of the unbalanced masses of the steam locomotive drivers, causing alternating forces of a sine form, coincides approximately with the natural frequency of the bridge. Hence, the fundamental consideration of dynamic design is based on this resonance case.

(2) Supplementary weights to equalize, partly, the reciprocating masses of steam locomotives passing at their highest allowable speed may cause centrifugal forces attaining, as a maximum, 15% of the vertical loads of drivers.⁶

⁵ "Vibration During Acceleration through a Critical Speed," by F. M. Lewis, *Transactions, A.S.M.E.*, 1932, APM-54-24-253.

⁷ "Report of the Bridge Stress Committee," by C. E. Inglis; published under the authority of His Majesty's Stationery Office, London, 1928, p. 201.

⁸ "Vibration of Locomotive Driving Wheels Caused by Unbalance," by O. J. Horger and C. W. Nelson, *Journal of Applied Mechanics*, Vol. 6, No. 4, December, 1939.

⁹ "Impact Stresses and Vibrations of Main Girders for Simple Span Railroad Bridges," by W. Hort, *Bautechnik*, 1928, No. 3, p. 37, and No. 4, p. 50.

¹⁰ "Mathematical Studies of Vibrations of Girders in Particular Cases," by R. Desprets; publication of the International Railroad Assoc., 1930 (Madrid) and the International Bridge Assoc., 1930 (Luettich).

(3) The damping capacities of bridges are taken from experimental values of published tests.^{11,12,13}

(4) Furthermore, experiments have shown that in the resonance case periodic forces may cause an amplifying factor of 16, with the loads acting in the middle of the span only. This decreases to 13, with the same loads moving.⁹ The reduction factor becomes, therefore, $13 : 16 = 0.81$. Hence, to simplify the design, the effect of the motion of the loads (the increasing influence in the center of the bridge and the decreasing influence at the ends) may be taken into consideration by this reduction factor.

(5) The main girders in the resonance case may oscillate in their fundamental vertical mode; for trusses, therefore, one impact value for all members is justified. Other periodic vibrations, such as horizontal or torsional vibrations, or even nonperiodic vibrations, are negligible in general. Theoretical derivations have proved that they do not exceed 7%, assuming the dynamic stresses due to the wheel effect at resonance to be 100%.⁹ Old locomotives of British railways produced a strong "hammer-blow" effect.⁷ The lifting of the driving wheels from the rails causes heavy shocks,⁸ which, however, are of primary importance for the rails and track girders only. Hence, nonperiodic vibrations, excited by direct impact on the track girders (rail joints or flat spots on wheels), which in certain cases might increase the impact value for single members of the main girders (overtones), are neglected.

(6) The dynamic design is developed only for statically determinate main girders and trusses of railroad bridges, with span lengths from 60 ft to 300 ft. The wheel effect does not come into consideration for span lengths less than 60 ft, since the critical speed is usually higher than the highest allowable train speed. The nomographic charts herein take into consideration all possible maximum and minimum values of the dynamic properties of loaded and unloaded bridges within the aforementioned span range.

COMPUTATION

Determination of the Natural Frequency of the Bridge.—The natural frequency ω_n of the bridge in cycles per second may be computed by the formula:¹³

$$\omega_n = \frac{\pi}{2} \sqrt{\frac{5}{384} \frac{g}{y}} \dots \dots \dots (1)$$

in which (see Appendix) g = acceleration due to gravity in feet per second per second and y = deflection of the bridge in feet.

In order to produce the deflection for the equivalent load, two extreme values must be taken into consideration: The minimum, being the dead load alone, and the maximum, being the dead load combined with the load of all vehicles on the bridge. For the moving loads, a uniformly distributed load

¹¹ "Dynamic Tests by Means of Induced Vibrations," by R. K. Bernhard, *Proceedings*, A.S.T.M., Vol. 37, Pt II, pp. 634-649.

¹² "Mechanical Vibrations of Bridges," by E. Homann and R. K. Bernhard, *Verkehrstechnische Lehrmittelgesellschaft*, Berlin, 1933.

¹³ "Dynamic Relations Between Moving Loads and Structures," by R. K. Bernhard, *Mechanical Engineering*, Vol. 60, No. 9, p. 697 et seq.; and "Dynamic Method of Investigating Stresses in Buildings," by R. K. Bernhard and W. Spaeth, *Der Stahlbau*, No. 6, 1926.

may be assumed. In most cases, the natural period with dead plus moving load is critical.

Determination of Critical Speed, Caused by Wheel Effect on the Bridge.—The critical speed in miles per hour may be determined by the equation:

$$V_c = 0.68 \pi D \omega_n \dots \dots \dots (2)$$

in which D = diameter in feet of the drivers of the passing locomotive.

The maximum and minimum values of V_c are determined by assuming the largest and smallest diameters of the drivers of the passing locomotives. A special case is the critical speed for wheels with flat spots on heavy freight cars with small wheel diameters. It is obvious that if the critical speed is higher than the highest allowable train speed, no further dynamic investigation is necessary.

Determination of the Number of Vertical Impulses Caused by Unbalanced Masses of the Passing Steam Locomotives.—The number of impulses, N_i , alternating with a sine form caused by unbalanced masses of a single driving wheel, may be determined by the formula:

$$N_i = \frac{L}{\pi D} \dots \dots \dots (3)$$

in which L = length of span in feet.

Determination of the Number of Vertical Impulses Necessary to Build up Oscillation.—The logarithmic damping decrement (θ) of bridges tested in recent years by the writer varies between 0.05 and 0.20.¹¹ It may be determined by recording power-input frequency diagrams (resonance curves) by means of mechanical oscillators. The width at the half point of the diagram, divided by the resonance frequency and multiplied by π , is equal to the damping decrement.¹² An average value of $\theta = 0.075$ may be suggested until more results are available. The nomographic charts, however, permit the use of maximum and minimum values.

The amplifying factor, F , for the resonance case (stable stage) and the number of periods, N_t , necessary to build up the oscillation (transient stage) may be determined with sufficient accuracy from the decrement, θ , by the following equations:¹³

$$F = \frac{\pi}{\theta} \dots \dots \dots (4a)$$

and

$$N_t = \frac{2.3}{\theta} \dots \dots \dots (4b)$$

In other words, F is the multiplication factor for obtaining the maximum influence for the dynamic forces in the resonance case.

The time to build up the oscillation in the resonance case is determined by the number of cycles that are necessary to reach the maximum excursion (stable stage). To attain this stable equilibrium (that is, constant amplitudes) an infinite time is necessary; to reach a finite time value, however, it may be

assumed that the equilibrium is reached as soon as the amplitudes have increased up to nine tenths of their final theoretical value.¹²

The computation is still accurate enough if, for the sake of simplicity, the amplitudes are assumed to increase from 0 to N_t cycles on an elliptic envelope

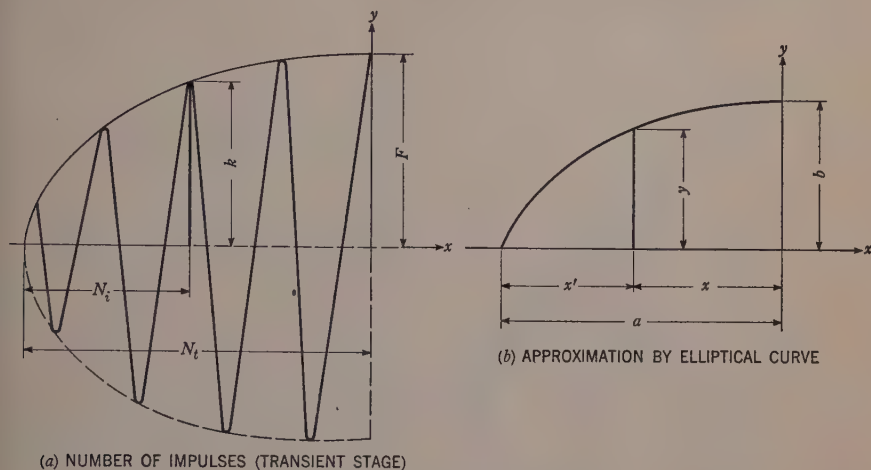


FIG. 1.—BUILDING UP VIBRATIONS

(Fig. 1(a)). The following simple formula can be developed from Fig. 1(b):

$$y = \frac{b}{a} \sqrt{a^2 - x^2} \dots \dots \dots (5a)$$

or, for $x' = a - x$:

$$y = \frac{b}{a} \sqrt{2ax' - (x')^2} \dots \dots \dots (5b)$$

Substitute $y = k$; $b = F$; $a = N_t$; and $x' = N_i$.

$$k = \frac{F}{N_t} \sqrt{2N_t N_i - N_i^2} \dots \dots \dots (6)$$

The value k represents that part of the amplifying factor which, after N_i cycles, is still effective.

Substituting for F and N_t the values of Eq. 4,

$$k = 1.365 \sqrt{2N_t N_i - N_i^2} \dots \dots \dots (7)$$

For most practical cases, the first part of the curve with small, and the last part with large, values of k do not occur; hence, the assumption of the elliptical envelope is accurate enough.

For further simplification, the increasing influence caused by the motion of the live loads near the center of the span, as well as the decreasing effect at the ends, may be taken into consideration by a reduction factor (experimental value) of 0.81:

$$k_w = 0.81 \times 1.365 \sqrt{2N_t N_i - N_i^2} = 1.11 \sqrt{2N_t N_i - N_i^2} \dots \dots (8)$$

Hence, k_w is equal to that part of the amplifying factor (F) caused by vertical forces of unbalanced locomotive drivers which is effective after N_i impulses. The symbol k_w may be called the amplifying number.

Determination of the Additional Vertical Forces.—The maximum value of the additional vertical forces can be determined by assuming that the load P_A imposed by the driving wheels (Fig. 2) does not exceed $0.15 P_A$ at the highest allowable speed.

The staggered arrangement of the counterweights, with a 90° phase difference on both sides of the locomotive, causes motions in two different planes. Two equal centrifugal forces, A_1 and B_1 , in a staggered arrangement of a 90° phase difference, are effective at the ends of each driving axle (Fig. 2). At the center of the driving axle add two equal forces in opposite directions—(A_2 and A_3) and (B_2 and B_3); the result is two staggered couples, composed of forces (A_1 and A_2) and (B_1 and B_2) with a 90° phase difference.

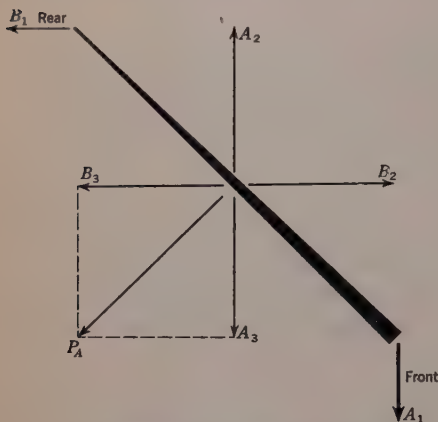


FIG. 2.—DISTRIBUTION OF UNBALANCED FORCES OF STEAM LOCOMOTIVE DRIVER AXLES

The remainder may be replaced by two staggered single forces, A_3 and B_3 , with a 90° phase difference, and a resultant force $P_A = A \sqrt{2} = 1.4 A$, assuming that $A_3 = B_3 = A$.

The first couple (forces A_1 and A_2) will excite the bridge to torsional vibrations; the second couple of forces (B_1 and B_2) will excite it to horizontal vibrations. Both couples may be neglected in the foregoing case. Only the vertical component $P_A = A \sqrt{2}$, alternating in a sine form and effective along the center line of the bridge, is to be considered. The numerical example has been computed, however, with the wheel load P_w , and not with $\frac{1}{2}$ times the vertical component P_A . This is to simplify any comparison with the static calculation, where often wheel loads of the theoretical train (standard specifications) are used. Finally, the free forces of the different driving wheels of one locomotive may be combined into one average force Z .

Knowing that the centrifugal forces increase with the square of the speed (angular velocity), the following equation is valid:

$$Z = 0.15 \times N_w \times P_A \left(\frac{V_c}{V_m} \right)^2 \dots\dots\dots (9)$$

in which V_m = highest allowable speed of the locomotive, and N_w = number of driving wheels of one locomotive.

The assumption is made that the case of two or even more locomotives on the bridge, with their counterweights in phase at the same time, is a rare exception which need not be taken into consideration; furthermore, the damping influence of adjacent cars, especially on longer bridges, may be neglected.

The effective vertical additional forces to build up the vibration are:

$$Z_w = k_w \times Z \dots \dots \dots (10)$$

Hence, Z_w is the maximum dynamic load on the bridge in tons, which becomes effective during the passage of the locomotive. In other words, it is the load that builds up the maximum oscillation under most unfavorable dynamic conditions.

Determination of Dynamic Impact.—The total stress is a combination of a static pre-stress and a superimposed dynamic stress (Fig. 3). The maximum value of the static pre-stress may be taken from the routine static computation,

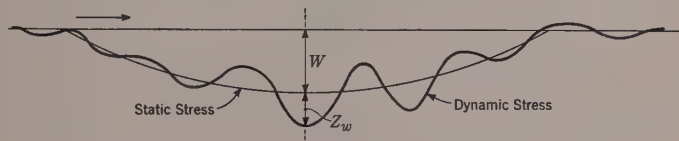


FIG. 3.—STATIC AND DYNAMIC STRESS DIAGRAM OF BRIDGE

but without impact, or from a design based upon the actual static loads of the previously chosen locomotive, whichever causes the most unfavorable conditions.

This static load may be equal to W tons. The dynamic load has already been determined by Eq. 10 to be Z_w tons. Eq. 11 gives the final dynamic impact number:

$$\phi = \frac{W + Z_w}{W} \dots \dots \dots (11)$$

Hence, to obtain the maximum static and dynamic stresses (total stress) for a certain vehicle (locomotive), the static loads must be multiplied by ϕ . Thus, for special cases, ϕ may replace the impact value in standard specifications.

NOMOGRAPHIC CHARTS

The nomographic charts in Figs. 4 and 5 make it convenient to determine the dynamic effect for an existing bridge, as well as for a new bridge design. Normally, space permitting, these charts should all be on the same sheet. In many cases it is sufficiently accurate to determine the final value with seven adjacent nomographic charts and one double scale. Networks of curves have the advantage of being independent of the quality of the drawing paper (no changing scale through humidity); however, they may cause confusion among different curves.

A special advantage of the nomographic chart is the easy determination of maximum and minimum values simultaneously. All scales and indicators, corresponding to Eqs. 1 to 11, could be drawn in logarithmic scales.

The seven nomographic charts in Figs. 4 and 5 may be constructed as a simple mechanism. The connecting lines can be drawn on celluloid strips, mounted with flexible links on the scales. After having determined the fundamental values on the proper scales, the result will be indicated automatically.

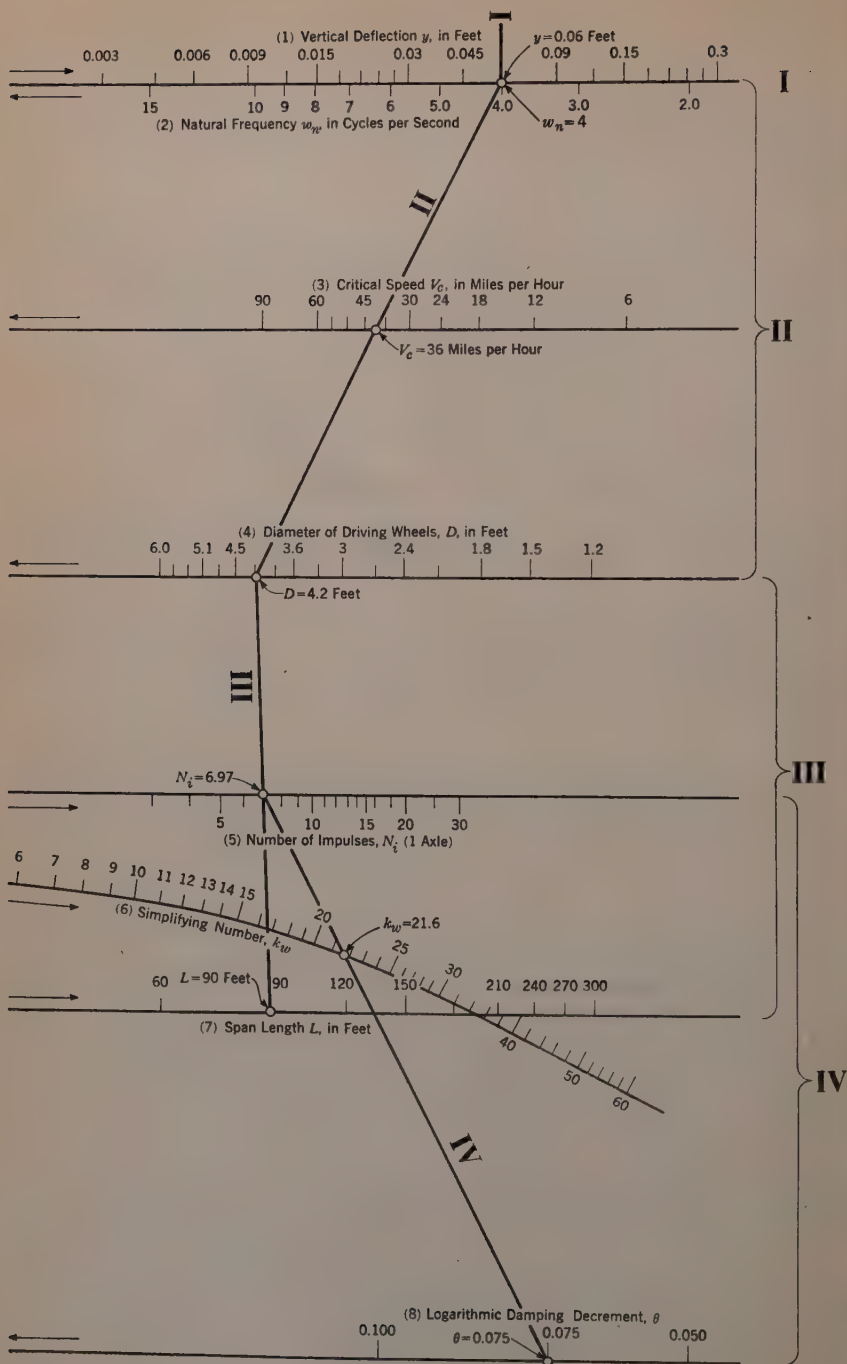


FIG. 4.—NOMOGRAPHS I TO IV

Certain cases may yield a dynamic impact number as great (for example) as $\phi = 3$, which is higher than that required by standard specifications. It must be borne in mind, however, that such a high dynamic impact number is not necessarily dangerous. Assume an allowable stress of 16,000 lb per sq in., an

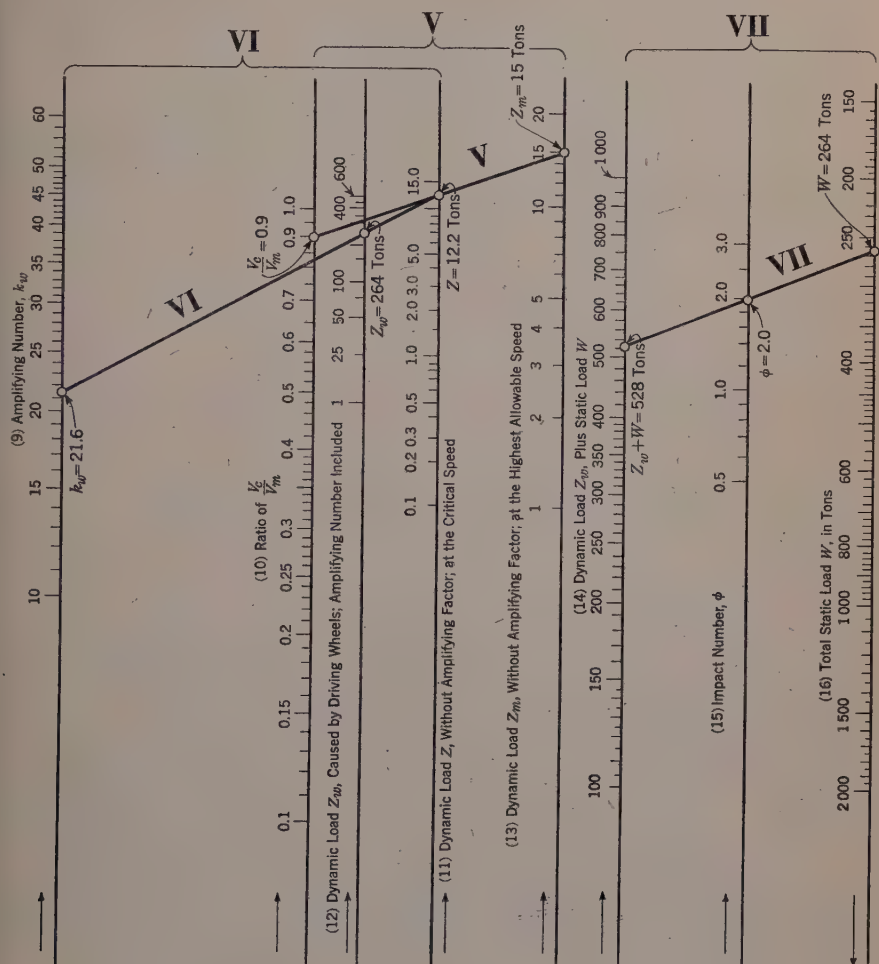


FIG. 5.—NOMOGRAPHS V TO VII

impact value, corresponding to the standard specification, of 1.4, and the same train loads as in the static design. The live load, including $I = 140\%$, may cause five eighths of the allowable stress ($\frac{5}{8} \times 16,000 = 10,000$ lb per sq in.). Hence, the combined stress of dead and live loads will be increased to 6,000

$$+ 10,000 \times \frac{3}{1.4} = 27,400 \text{ lb per sq in.}$$

This value is lower than the yield strength, but might be higher than the endurance limit of the steel (punched or

drilled structural steel). However, the bridge will stand the traffic for many years without failure, especially if the traffic is light (effect of repose).

NUMERICAL EXAMPLE

The complete dynamic investigation of a bridge consists in drawing six straight lines in the corresponding nomographic charts I to VII (Figs. 4 and 5).

The following example will serve as an illustration: The dynamic number ϕ is to be determined for a single-track bridge of 90-ft span length, with a deflection y of 0.06 ft, and a damping decrement θ of 0.075. The structure may have an open trackway and welded rail joints. The passing freight locomotive may have five driving wheels of diameter $D = 4.2$ ft, a wheel load of 20 tons, and a speed limit of 40 miles per hr. In seven steps:

(1) Scales 1 and 2, Fig. 4, indicate, for $y = 0.06$ ft, the natural period $\omega_n = 4$.

(2) Drawing the connecting line II in the nomographic chart II ($\omega_n = 4$ and the wheel diameter $D = 4.2$ ft), the indicator shows for the critical speed in scale 3: $V_c = 36$ miles per hr.

(3) Nomographic chart III yields, with the connecting line III ($D = 4.2$ ft and span length $L = 90$ ft), the number of impulses $N_i = 6.97$ on the indicator (scale 5).

(4) Nomographic chart IV indicates, with the connecting line IV ($N_i = 6.97$ and the damping decrement $\theta = 0.075$), an amplifying number: $k_w = 21.6$ (scale 6).

(5) Drawing the connecting line V in Fig. 5 (dynamic load at maximum speed $Z_m = 0.15 \times 5 \times 20 = 15$ tons and $\frac{V_c}{V_m} = 0.9$), the indicator shows for the dynamic load: At critical speed $Z = 12.2$ (scale 11).

(6) Nomographic chart VI gives, with the connecting line VI ($Z = 12.2$ and $k_w = 21.6$), the amplifying number included, the dynamic load: $Z_w = 264$ tons (scale 12).

(7) The final answer in chart VII is determined by drawing the connecting line VII ($W_{90 \text{ ft}} = 264$ tons and $W + Z_w = 528$ tons), yielding the impact number: $\phi = 2.0$ on the indicator (scale 15).

Result.—The impact increment, corresponding to the old American Railway Engineering Association (A.R.E.A.) specifications, would be 1.79 and is smaller than $\phi = 2.0$.

The maximum stress caused by the live load, including $I = 179\%$, may be five eighths of the total stress $= \frac{5}{8} \times 16,000 = 10,000$ lb per sq in., or, taking ϕ into consideration, the stress is $10,000 \times \frac{2.0}{1.79} = 11,200$ lb per sq in. The stress caused by the dead load is three eighths of the total load; hence, $\frac{3}{8} \times 16,000 = 6,000$ lb per sq in., and the total stress, therefore, becomes, 17,200 lb per sq in.

The dynamic investigation proves in this case that the endurance limit for punched or drilled structural steel, which may be assumed as one fourth of the

ultimate tensile strength ($0.25 \times 65,000 = 16,250$ lb per sq in.), is about 1,000 lb per sq in. smaller than the maximum dynamic stress.

CONCLUSIONS

The example presented herein can be supplemented to include span lengths from 60 ft to 300 ft and three different and comparatively light two-cylinder steam locomotives. The three locomotives are: One high-speed locomotive with three pairs of driving wheels of 6.9-ft diameter, 20-ton wheel load, and an allowable speed of 75 miles per hr; one medium-speed locomotive with three pairs of driving wheels of 5-ft diameter, 15-ton wheel load, and an allowable maximum speed of 55 miles per hr; and the aforementioned freight locomotive. The freight locomotive, with its five pairs of small drivers and relatively slow allowable speed, but comparatively large wheel loads, causes the highest dynamic impact.

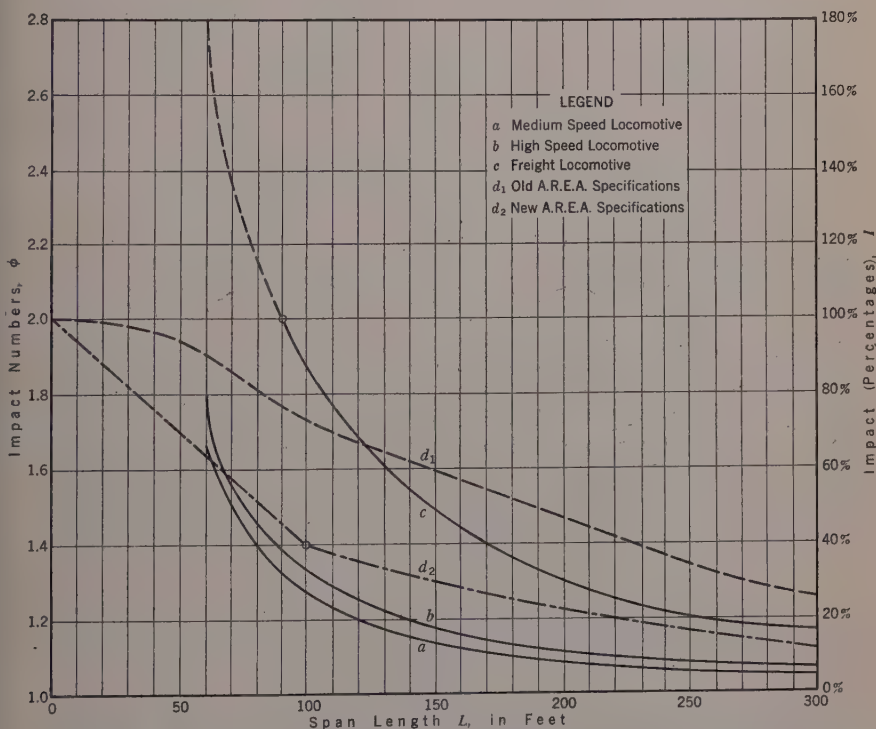


FIG. 6.—COMPARISON BETWEEN IMPACT NUMBERS ϕ OF THREE DIFFERENT TYPES OF LOCOMOTIVES AND A.R.E.A. SPECIFICATIONS

This result has often been verified in field tests.^{5,7,9,13} It is not the two-cylinder high-speed and medium-speed steam locomotives, but nearly always two-cylinder freight engines with small driving wheels and relatively slow allowable speeds, which cause the greatest dynamic stresses. The numerous impulses on the bridge, and the centrifugal forces growing rapidly to their maximum value, are predominant.

For high-speed passenger service, steam locomotives with three or more cylinders, electric engines, or light-weight motor-driven engines may come more and more into favor. For freight traffic, the heavy two-cylinder locomotive with its high impact influence cannot be replaced so easily.

Fig. 6 shows clearly the increasing value of ϕ for bridges with shorter span lengths (60 ft to 120 ft). Similar results are reported by other investigators;^{5,7} for relatively long bridges (more than 300 ft), as well as for short bridges (less than 90 ft), the dynamic influence caused by resonance between the unbalanced masses of steam locomotive parts and the natural frequency of the bridge decreases considerably. The tendency of the ϕ -curves, which decrease gradually with the span length (over 90 ft), is the same as in the impact curves of standard specification.

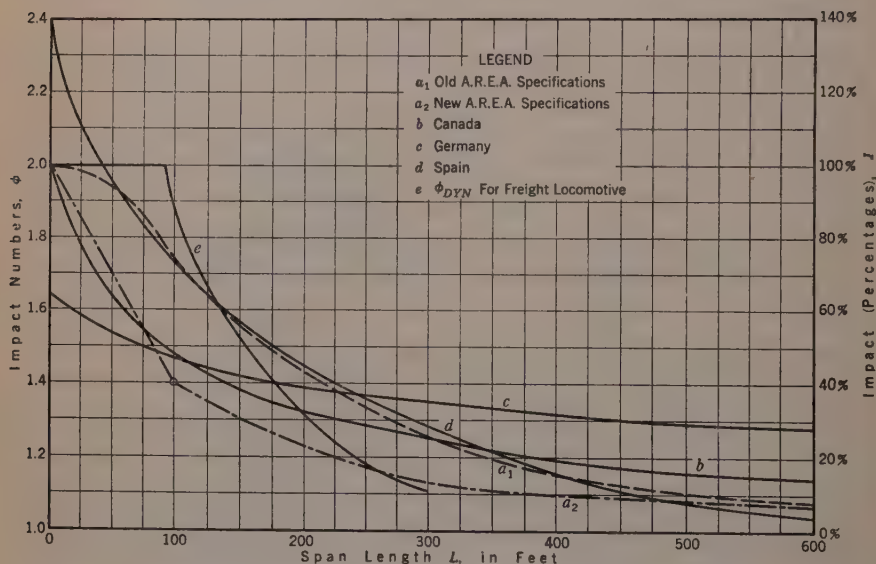


FIG. 7.—COMPARISON BETWEEN IMPACT NUMBERS ϕ AND IMPACT IN PERCENTAGE FOR BRIDGES IN VARIOUS COUNTRIES

Fig. 7 represents a comparison between the two A.R.E.A. formulas:⁵ The old formula—

$$I = \frac{30,000}{30,000 + L^2} \dots \dots \dots (12)$$

and the new formulas—

$$I = 100 - 0.60 L \quad (L \leq 100 \text{ ft}) \dots \dots \dots (13a)$$

and

$$I = \frac{1,800}{L - 40} + 10 \quad (L \geq 100 \text{ ft}) \dots \dots \dots (13b)$$

as well as the impact specifications of other countries.

It must be borne in mind, however, that, for span lengths less than approximately 90 ft, the dynamic influence will increase considerably for other reasons. The wheel effect is no longer important, because the critical speed becomes higher than the speed limit. On the other hand, the direct shocks are more effective since, for example, the distance from the exciting (nonperiodic) forces on the rail to the shorter girders, especially track girders, becomes smaller.

For span lengths between 200 ft and 300 ft, the impact number ϕ , as computed dynamically, follows the new A.R.E.A. specification, even under the most unfavorable dynamic conditions. In case of span lengths less than 200 ft, the values of ϕ may become higher than those of the new A.R.E.A. formula, although following rather closely the old A.R.E.A. specifications.

SUMMARY

(1) A dynamic stress analysis of railroad bridges becomes advisable in certain cases, especially if the impact value required by the standard specifications is smaller than the actual value.

(2) A simple and approximate dynamic analysis is possible for a considerable number of medium-size bridges by the use of nomographic charts which may avoid laborious computations.

(3) A dynamic analysis is possible only if certain dynamic qualities of the bridge are known. Furthermore, the dynamic properties of the different moving loads must be taken into consideration separately for each type of passing vehicle. The largest dynamic stresses are caused in many cases by relatively slow freight locomotives and not by high-speed or medium-speed engines.

(4) The method outlined in this paper should be verified further by tests on old and new bridges; that is, it should be clearly understood that the assumptions made in simplifying the dynamic computation require experimental checking. The dynamic properties of bridges, as well as of vehicles, both while standing and while moving, should be investigated—that is, their natural frequency, damping capacity, vibrating masses, deflection and stress diagrams, dynamic wheel-load diagrams, and the influence of the motion of the loads.

APPENDIX

NOTATION

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials compiled by a Committee of the American Standards Association¹⁴ with Society representation, and approved by the Association in 1932:

A = centrifugal force vector in Fig. 2;

a = major axis of ellipse;

¹⁴ ASA-Z10a-1932.

B = centrifugal force vector in Fig. 2;

b = minor axis of ellipse;

D = diameter of driving wheels of locomotive;

F = amplifying factor for resonance case;

g = acceleration due to gravity;

I = impact, in percentage;

k = substitution constant (see Eq. 6):

k_w = amplifying number (see Eq. 8);

L = length of span, in feet;

N = number:

N_i = number of impulses;

N_t = number of periods necessary to build up oscillation;

N_w = number of driving wheels of one locomotive;

P = concentrated load:

P_A = axle load = load imposed by locomotive drivers;

P_a = wheel load;

V = velocity; speed in miles per hour:

V_c = critical speed;

V_m = highest allowable speed of locomotive;

W = total static load; weight;

x = abscissas;

y = ordinate = vertical deflection of bridge;

Z = dynamic load at critical speed:

Z_m = maximum dynamic load without amplification factor;

Z_w = maximum dynamic load including amplification factor;

θ = logarithmic damping decrement;

ϕ = impact number;

ω = frequency:

ω_n = natural frequency.

EXPERIENCES IN OPERATING A CHEMICAL-MECHANICAL SEWAGE TREATMENT PLANT

BY GEORGE J. SCHROEPFER,¹ ASSOC. M. AM. SOC. C. E.

SYNOPSIS

The Minneapolis-St. Paul (Minn.) sewage treatment plant began operating on June 1, 1938. In reporting the experiences of the first two years of operation, data are included on the performance of the plant and the cost of operation and maintenance, as well as on the improvement of the Mississippi River which has resulted from its operation. Some of the problems that arose in early operation, and the methods used to overcome them, are discussed, as well as other improvement changes which have been incorporated to simplify operation or effect economies. It is hoped that such statements will encourage the presentation of data on similar experiences at other plants, as well as make available for the benefit of others the experiences at this plant.

DEGREE OF TREATMENT

Because of its importance to a proper consideration of the results of operation and the statements concerning experiences made herein, as well as because it is the most important factor in the consideration of any pollution abatement project, some space is devoted to the degree of treatment provided in this project.

The Mississippi River at St. Paul fluctuates widely, both as to discharge (which varies from a minimum of 864 cu ft per sec to a maximum estimated flow of 107,000 cu ft per sec with an average annual flow of 8,800 cu ft per sec) and in the physical, chemical, and biological factors that affect its capacity to receive and assimilate pollution. This condition, and the fact that the Twin City sewage formerly discharged, and now the effluent of the treatment plant discharges, into pools created by the canalization project, affects to a greater or lesser extent the type of treatment selected, the degree, and the duration for which various degrees of treatment must be maintained. In the studies of treatment necessary, the information collected on river conditions by various

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by May 15, 1941.

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authorities over the relatively long period of practically continuous record from June, 1926, to October, 1934, when the decision on type of treatment was made, was of prime importance in adjusting proposed sewage treatment to river requirements. This record of analytical data has been continued, so that at the present time (1941) it covers a period of fifteen consecutive years.

The pollution load on the river is predicted to reach a 1,300,000 population-equivalent for the Twin City metropolitan area by 1945, exclusive of South St. Paul and Newport, Minn., from a tributary population of 910,000. Computations using the 42-yr period of river record, from 1892 to 1933, inclusive, indicated, for the foregoing pollution load, (1) that sedimentation alone (36% B.O.D. reduction—that is, 5-day biochemical oxygen demand) would suffice to meet river standards as interpreted herein, except for 12.5% of the time; (2) that sedimentation plus effluent filtration (45% B.O.D. reduction) would be sufficient except for about 8% of the time; and (3) that sedimentation and effluent filtration, plus chemical treatment (to 70% B.O.D. removal), would maintain standards except for 1.5% of the time.

A 5-day B.O.D. removal of 70% was adopted as the degree of treatment necessary to meet all reasonable river requirements satisfactorily. A variety of treatment processes were investigated progressively in the period from 1927 to 1934 to provide this removal. As a result, sedimentation plus effluent filtration, augmented at times of critical river flow by chemical treatment, was adopted as the process which would meet the river requirements most satisfactorily and economically.

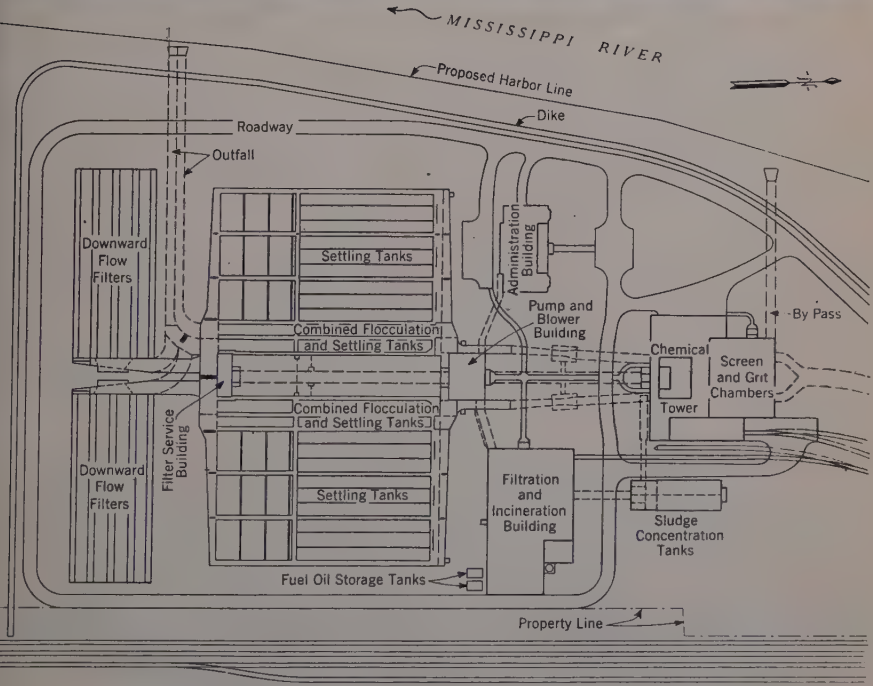
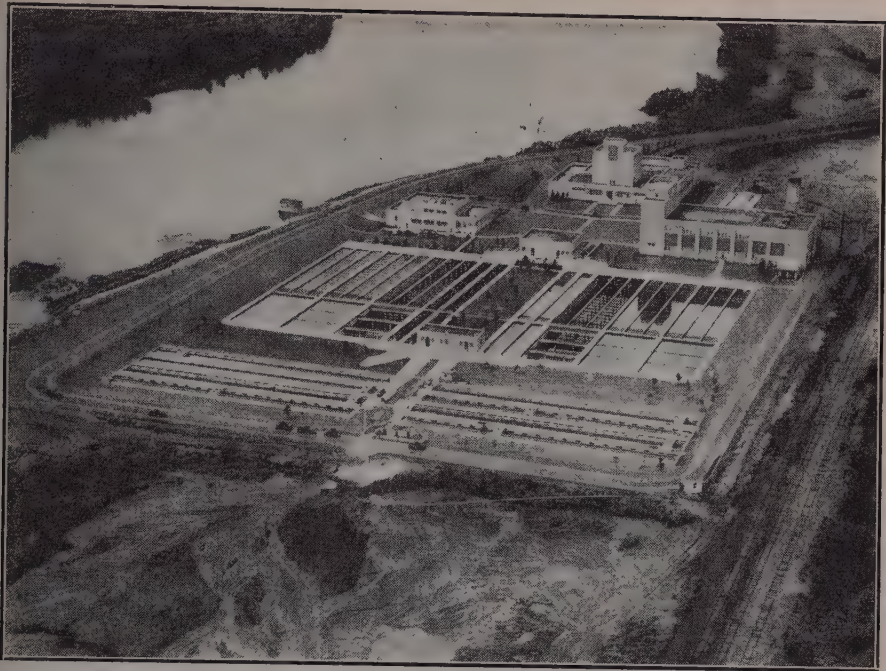
A clearer conception of the variation in treatment requirements can be had from the following: Considering river flows in the period from 1892 to 1933, and the expected 1945 pollution load, chemical treatment would be required during only 40 months of the total of 504 months, and the average B.O.D. removal required during the 40 months that chemical treatment was necessary was approximately 58%. Expressed in another way, sedimentation plus effluent filtration would be sufficient treatment based on past river flows for a continuous period of fourteen years; and then, after a 2-yr period when chemical treatment would be required, preliminary treatment again would be sufficient for another 12-yr period. Sedimentation plus effluent filtration would be sufficient, and no chemical treatment would be required for thirty years out of the 42-yr period of record.

GENERAL PLANT DESCRIPTION

The treatment plant² was designed for an average daily flow of 134 mgd expected from a tributary population of 910,000 in 1945. The intercepting sewer leading to the plant has a capacity of 610 mgd, which flow will be reached on infrequent occasions during storms.

Described briefly, the sewage treatment plant consists of screen and grit chambers, flocculating and settling tanks, and effluent filters, with provision for chlorination and chemical treatment (see Fig. 1). The sludge disposal process consists of concentration tanks, vacuum filters, and incinerators. A

² For a more detailed description of the treatment process see "Layout and Plan of Operation of the Minneapolis-St. Paul Sewage Treatment Plant," by George J. Schroepfer, *Sewage Works Journal*, Vol. 9, 1937, p. 913.



ST. PAUL BRIDGE & TERMINAL RY. CO. LEASED TO CHICAGO GREAT WESTERN RY CO

FIG. 1.—CHEMICAL TREATMENT PLANT, MINNEAPOLIS-ST. PAUL SANITARY DISTRICT

distinct effort was made in the design of the plant to provide a maximum of flexibility and dependability. From this standpoint, and also to secure information on the relative economy and effectiveness of various processes and chemicals, the entire plant, from the point of entrance of raw sewage to the discharge of treated effluent into the river, is divided into two distinct and separately operated divisions. Similar flexibility is obtainable in the sludge disposal processes. Features are incorporated in the design for treatment by a variety of processes.

Screen and Grit Chambers.—The intercepting sewer conducting the sewage to the screen and grit chambers is a double-barrel section, each of which is 9.5 ft wide and 10 ft deep, with a total design capacity of 610 mgd. From each intercepting sewer barrel the sewage passes through the two separate batteries of the screen and grit chambers, the individual units of which are described briefly, as follows:

Coarse Bar Racks.—Two hand-raked racks in channels 10 ft wide with a maximum flow depth of 10 ft and with clear openings of 6 in.

Bar Screens.—Four mechanically and automatically cleaned bar screens with one-inch clear openings, each 10 ft wide, with a maximum sewage depth of 8.7 ft.

Grit Chambers.—Eight grit chambers, 62.75 ft long, from the inlet gate to the outlet gate. The width of the chambers is 12 ft and the depth of flow varies from 4.5 ft to 9.0 ft. The velocity of flow varies from 0.45 to 1.18 ft per sec.

Auxiliary Units.—Provision was made for two screenings shredders, each with a capacity of 3,500 lb per hr; a grit elevator and hopper for discharge of grit into trucks or freight cars; bins for emergency grit and screenings storage; and a 5-ton crane equipped with a $\frac{3}{4}$ -yd bucket to be used for handling grit and screenings from these bins, as well as for several other purposes.

Chemical Control Building.—The chemical control building is a part of the superstructure over the screen and grit chamber. It houses the dry chemical feeders, chlorinators, pneumatic conveying equipment, and storage bins, described in more detail as follows:

Chemical Feeders.—Three dry chemical feeders of the gravimetric type, each having a feeding rate infinitely variable between 400 and 2,500 lb per hr.

Lime Feeders.—Two lime feeders, each of which has a capacity range from 400 to 2,500 lb per hr, equipped with lime slakers.

Chlorinators.—Five solution feed chlorinators, each with a capacity range from 850 to 6,000 lb of chlorine per twenty-four hours.

Pneumatic Conveying Equipment.—Pneumatic conveying equipment with a capacity of 10 tons per hr for unloading and elevating dry chemicals.

Chemical Storage Bins.—A total of ten concrete storage bins, each having a capacity of approximately 2,550 cu ft.

Flocculating and Settling Tanks.—Included under this heading are the venturi meters, combination flocculation and settling tanks, and the settling tanks.

Venturi Meters.—Each of the two venturi meters has an inlet diameter of 9 ft and a throat diameter of 5 ft, and is designed for a minimum flow of 43 mgd and a maximum flow of 350 mgd.

Combination Flocculating and Settling Tanks.—Although they are designated as flocculating tanks, these units have been arranged for multiple functioning, including their use as a by-pass channel, settling tanks, post-chlorination contact tanks, and grease separation tanks, in addition to flocculation with and without chemicals. The tanks proper consist of two units, one adjacent to each battery of settling tanks, each unit consisting of two passes which are each 17.75 ft wide and 290 ft long with an average water depth of 15.5 ft. The tanks provide a detention period of 24.5 min at the average flow of 134 mgd. When used as settling tanks, they therefore add 24.0 min to the sedimentation period.

Settling Tanks.—The two batteries of settling tanks consist of three units, each 56 ft wide, 290 ft long, and with an average water depth of 15.5 ft. Each tank has three passes with a width of 18.0 ft. The detention period in the settling tanks at the average flow of 134 mgd is two hours. The tanks are provided with sludge removal and skimming mechanisms. A rather comprehensive system of effluent weirs has been provided, having a total length of 480 ft for each tank and providing 0.047 million gal per ft of weir length at average flow.

Pump and Blower Building.—In this building are located various services used in connection with flocculation and sedimentation, including two 300-gal-per-min sump-drainage and under-drainage pumps; two 200-gal-per-min scum pumps; eight 210-gal-per-min triplex displacement sludge pumps; two 500-gal-per-min centrifugal sludge pumps for peak and emergency purposes; one 2,500-gal-per-min tank dewatering pump; and three blowers, one 1,200 cu ft per min, one 1,800 cu ft per min, and one 2,400 cu ft per min, along with the switchboard for control of the various equipment, a small office, and various minor miscellaneous services.

Effluent Filters.—The effluent filters are of the downward-flow magnetite sand type, are eight in number, and are divided into two batteries. The total filter area is 31,200 sq ft, which provides a filter rate of 3 gal per sq ft per min at average flow of 134 mgd. Each filter bed is 16 ft wide and 244 ft long.

Filter Service Building.—In this building are located three 1,500-gal-per-min wash-water pumps, two 300-gal-per-min sump pumps, one pump of 300-gal-per-min capacity, and two pumps of 450-gal-per-min capacity each to pump plant effluent for use in sluicing ash or dissolving and transporting chemicals; and three motor generator sets of 50-hp capacity each.

SLUDGE DISPOSAL

General.—The sludge removed from the settling tanks is pumped to the sludge concentration tanks. After concentration the sludge is conducted to the conditioning tanks, thence to vacuum filters, and finally to the incinerators.

Concentration Tanks.—Two such tanks, each 95 ft long, 18 ft wide, and with a total water depth of 24 ft (effective capacity of 41,000 cu ft each), are provided, with sludge collector mechanisms and means for removing scum. The number of tanks in use, the depth of sludge, and therefore the period of storage are varied with the sludge load.

Filtration and Incineration Building.—In this building, in addition to the vacuum filters, incinerators and their appurtenances, the machine shop, boiler room, and electrical control equipment are located.

Vacuum Filters.—The sludge from the concentration tanks is fed to either of two bucket elevators which discharge into three air-agitated conditioning tanks. From these tanks the sludge discharges by gravity into six vacuum filters arranged in two batteries. Each filter is 11.5 ft in diameter and 14.0 ft long, and has an effective filter area of 500 sq ft. Among the main appurtenances of the vacuum filters may be mentioned three vacuum pumps, each with a capacity of 1,850 cu ft per min; three blowers, each with a capacity of 800 cu ft per min; three filtrate pumps, each with a capacity of 250 gal per min; and two belt conveyers with weighing scales, each having a capacity of from 2 to 30 tons per hr.

Incinerators.—Three incinerators of the multiple-hearth type have an outside diameter of 22.25 ft, with eight hearths, each incinerator having a nominal capacity of 60 tons of dry solids. The incinerator ash is discharged into hoppers, from which it is sluiced to ash pumps, and thence is discharged to the dump. The main appurtenances of the incinerators are three plate-type preheaters, three hot-air fans each with a capacity of about 11,000 cu ft per min, and three hot-gas fans each with a capacity of about 36,000 cu ft per min.

Chemical Handling and Feeding.—A pneumatic conveying system with a capacity of 6 tons per hr has been provided for unloading dry chemicals and discharging them into storage bins.

Chemical feed equipment was installed for feeding lime with either chlorinated copperas, ferric sulfate, or ferric chloride, the use of which will depend on the efficiency and market prices of the various chemicals from time to time. Two gravimetric lime feeders and slakers with a capacity range from 200 to 1,200 lb per hr each, as well as two copperas or ferric sulfate feeders with a capacity range from 100 to 600 lb per hr each, have been installed. In addition to the aforementioned feeders, two solution feeders for ferric chloride have been provided, each with a capacity range from 12 to 120 gal per hr.

PLANT OPERATION

Operation of Plant Started.—Shortly after the dedication of the treatment plant on May 16, 1938, preliminary operation and testing of the equipment were begun. On June 1 the first sewage was turned into the intercepting sewers and into the plant, and by July 15, 1938, practically all of the sewage of the Twin Cities was conducted to the plant.

The plant was placed in operation under a planned and definite schedule which called for a gradual starting of the various units. As already stated, the plant is divided into two separately operated halves or batteries that can be operated independently of each other. Advantage was taken of this fact in starting the plant. The various plant units were placed in operation one battery at a time. The schedule of placing the plant in operation (which was carefully adhered to) was as follows:

- June 1—One battery of screen and grit chambers placed in operation.
- June 7—Operation of second battery begun.
- June 9—One battery of settling tanks started.
- July 2—Vacuum filtration of sludge started and filter cake hauled to dump.
- July 6—Settled effluent turned into magnetite filters.
- July 7—Second battery of settling tanks placed in service.
- July 21—Incineration of sludge begun.

Operation of the various units was begun without unusual difficulty except as might be expected with mechanical equipment of the type required in a large plant of this type. The manufacturers of the magnetite effluent filters and the incinerators experienced the greatest difficulties in placing their equipment in satisfactory operation. Some of these difficulties are discussed in more detail subsequently.

Continuous 24-hr operation of the plant was begun on July 20. From then on all the equipment that had been accepted by the Minneapolis-St. Paul Sanitary District performed in an acceptable manner. However, because of the manufacturer's difficulties with the incinerators, it was necessary to bypass varying quantities of sewage from time to time, depending on the quantity of sludge that could be incinerated. After October 3 no sewage was by-passed at the plant with the exception of excess storm flows, the excess sludge that could not be handled by the incinerators being pumped directly to the river. It was not until the latter part of November, 1938, that the incinerators were capable of incinerating all the sludge produced at the plant.

With the large volume of equipment in a modern sewage treatment plant, some of which has been used in this field only a few years, the process of placing a large plant in operation is rather involved, especially from a mechanical standpoint. The writer wishes to record with satisfaction the fine cooperation accorded by manufacturers of various equipment. It is gratifying to note that, although extensive guarantee and maintenance bonds of various types were provided for in the mechanical contracts, it has not been necessary to invoke any of their compliance provisions.

General.—The sewage flow arriving at the plant was less than the estimated flow of 120 mgd for the year 1938, averaging 98.8 mgd for the period from July 1, 1938, to May 31, 1940, inclusive. The strength of the incoming sewage, however, was somewhat greater than was anticipated, averaging 275 ppm on a suspended-solids basis, and 210 ppm on a 5-day B.O.D. basis. This compares with expected strengths of 185 ppm and 200 ppm, respectively. The increased strength on a suspended-solids basis places a greater load on the treatment plant, especially on the sludge disposal processes, than was anticipated.

Up to May 31, 1940, a total of 66,864.1 million gal of sewage was treated by the plant, from which a total of 79,944 cu ft of screenings and 501,278 cu ft of grit was removed. In addition, solids equivalent to a total of 183,497.6 tons of filter cake were removed and incinerated. More detailed operating data are given in Table 1.

As already stated, the plant was designed so that treatment by a variety of processes can be provided to meet varying river requirements. Operation

TABLE 1.—ANALYTICAL AND OPERATING

Month	AVERAGE DAILY SEWAGE FLOW IN MILLIONS OF GAL			Screenings, in cu ft per million gal	Grit, in cu ft per million gal	B.O.D.				
	Total	Minneapolis	St. Paul (by difference)			PPM		Percentage of Removal		
						Raw sewage	Settled effluent	Settling tanks	Effluent filters	Total

(a) SIX MONTHS IN 1938										
July ^a	93.6	70.4	23.2			170	105	35.9		
August ^b	71.357	65.776		1.15	6.3	180	115	35.8		
September.....	87.397	65.966	21.4	1.05	6.3	175	120	29.3		
October.....	89.819	64.322	25.5	0.98	7.2	210	130	38.6		
November.....	92.835			1.15	6.9	230	160	31.3		
December.....	98.59	67.133	31.5	1.00	6.2	245	135	44.7		
Average.....	88.9	66.6	22.3	1.07	6.6	200	130	36.0		

(b) TWELVE MONTHS IN 1939										
January.....	99.266	73.814	25.4	1.12	6.0	230	130	42.5		42.5
February.....	96.481	69.481	26.9	1.17	4.8	235	155	34.3		34.3
March.....	114.543	81.288	33.2	1.24	7.5	200	135	31.9		31.9
April.....	97.532	69.568	28.8	1.61	8.75	210	130	38.5		38.5
May.....	100.383	73.788	26.6	1.76	10.65	205	130	35.0		35.0
June ^c	110.836	78.629	33.0	1.28	13.9	170	91	45.7		45.7
July ^d	107.105	74.926	32.2	1.03	12.4	165	98	40.2		40.2
August.....	108.150	75.365	32.8	1.04	9.39	170	105	39.6		39.6
September.....	108.389	74.122	36.8	1.17	10.1	190	115	39.7		39.7
October.....	106.157			1.52	9.3	220	130	40.2		40.2
November ^e	94.112			1.25	7.8	250	145	40.0	2.8	42.8
December.....	92.286	71.009	21.3	1.11	4.7	240	145	39.6	2.6	42.2
Average.....	102.937	74.199	29.7	1.28	8.77	205	125	38.9		39.4

(c) FIVE MONTHS IN 1940										
January.....	91.321			0.88	3.1	240	140	41.8		
February.....	90.164			0.96	3.4	235	140	39.4	0.7	40.1
March.....	109.314	77.051	32.3	1.08	6.5	200	130	33.8	1.6	35.4
April.....	108.212	67.490	38.1	1.57	7.1	220	140	35.8	0.1	35.9
May.....	104.898	66.031	38.9	1.08	5.4	220	135	39.0	0.9	39.9
Average.....	100.732	70.191	36.4	1.11	5.1	225	135	38.0	0.8	37.8

^a Plant placed in partial operation, 24-hr basis, July 20. ^b Considerable by-passing during August. ^c Minneapolis Trial chemical treatment using ferric sulfate. ^d 27 days of operation. ^e Gross removals, 70.5% and 77.5%, were used in the average for the year. ^f Gross removals, 70.0% and 71.2%, were actually reduced to 59.5% and 60.7%.

to date (1941) indicates that with all units in operation the following removals can reasonably be expected:

Treatment process	5-day B.O.D.	Suspended solids
Sedimentation only.....	40 to 45%	70 to 75%
Flocculation plus sedimentation....	45 to 50%	75 to 80%
Flocculation and sedimentation, plus effluent filtration.....	50 to 55%	80 to 85%
Flocculation with chemicals, sedimentation, and effluent filtration (dependent upon quantities of chemicals).....	55 to 70%	85 to 95%

DATA ON SEWAGE TREATMENT

SUSPENDED SOLIDS					SETTLABLE SOLIDS					ALKALINITY				pH		TEMPERATURE, IN DEGREES F		Detention period in tanks, in hr
PPM		Percentage of Removal			Ml per Liter		Percentage of Removal			Raw Sewage		Settled Effluent						
Raw sewage	Settled effluent	Settling tanks	Effluent filters	Totals including screenings and grit	Raw sewage	Settled effluent	Settling tanks	Effluent filters	Total	Methyl orange	Phenolphthalein	Methyl orange	Phenolphthalein	Raw sewage	Settled sewage	Raw sewage	Effluent	
(a) SIX MONTHS IN 1938																		
235	62	72.9	..	..	7.0	0.3	92.4	..	..	270	0	265	0	7.3	7.3	68	68	2.4
235	75	66.5	..	..	5.9	0.5	91.2	..	..	280	0	270	0	7.2	7.3	70	70	1.5
225	81	62.1	..	..	6.0	1.2	80.0	..	..	290	0	285	0	7.1	7.2	66	..	1.2
255	67	73.0	..	..	6.9	0.6	91.9	..	..	320	0	310	0	7.2	7.3	65	..	1.85
250	83	65.9	..	..	6.7	0.6	90.9	..	..	330	0	325	0	7.2	7.3	61	..	2.45
255	67	73.0	..	..	7.2	0.4	94.3	..	..	335	0	325	0	7.1	7.3	58	57	3.5
240	72	68.8	..	..	6.6	0.6	90.0	..	..	305	0	295	0	7.2	7.3	65	..	2.15

(b) TWELVE MONTHS IN 1939

245	74	70.0	..	73.6	7.0	0.7	88.4	..	88.4	325	0	315	0	7.3	7.5	56	56	2.1
280	105	61.2	..	67.0	7.8	1.7	77.3	..	77.3	330	0	320	0	7.4	7.5	56	55	0.9
360	125	63.3	..	70.8	6.7	1.9	68.4	..	68.4	260	0	255	0	7.4	7.5	54	52	1.2
330	97	69.0	..	75.9	7.2	1.2	83.6	..	83.6	220	0	210	0	7.4	7.5	55	54	1.3
275	85	63.5	..	75.6	6.1	1.3	77.6	..	77.6	270	0	260	0	7.4	7.5	61	60	1.0
310	63	77.9	..	84.6	6.5	0.8	87.5	..	87.5	270	0	260	0	7.5	7.5	65	64	1.5
285	69	74.4	..	82.5	6.3	0.9	86.5	..	86.5	240	0	235	0	7.5	7.5	69	67	1.2
245	75	67.7	..	78.0	6.7	0.9	87.1	..	87.1	265	0	255	0	7.4	7.5	69	68	1.0
250	77	68.4	..	77.9	6.9	0.9	88.0	..	88.0	285	0	275	0	7.5	7.5	67	66	1.0
280	77	72.3	..	78.1	7.8	0.8	89.8	..	89.8	290	0	285	0	7.5	7.5	65	64	1.1
300	88	70.3	2.5	76.2	8.8	0.9	90.4	0.7	91.1	300	0	300	0	7.4	7.5	61	59	1.5
280	81	60.6 ^A	4.3	68.6 ^A	7.5	0.5	93.7	1.3	95.0	300	0	300	0	7.5	7.5	59	57	1.5
285	85	68.2	..	75.7	7.1	1.0	84.9	..	85.0	280	0	275	0	7.4	7.5	61	60	1.3

(c) FIVE MONTHS IN 1940

265	78	59.5 ^s	60.7 ^s	64.0	7.7	0.5	94.3	0.2	94.5	305	0	300	0	7.5	7.5	56	55	1.5
310	75	74.7	77.8	77.8	7.9	0.3	86.2	0.3	86.5	290	0	285	0	7.5	7.5	56	54	2.4
310	100	65.0	66.5	70.0	6.1	0.8	95.6	0.6	86.1	220	0	220	0	7.4	7.5	52	50	1.2
310	100	66.9	66.9	73.4	6.3	1.1	83.6	0.5	84.1	245	0	240	0	7.4	7.5	53	53	1.1
300	76	73.8	75.4	78.5	7.0	0.4	94.4	0.7	95.1	250	0	245	0	7.5	7.5	58	58	1.3
300	86	68.0	69.0	73.1	7.0	0.6	90.8	0.5	91.3	260	0	260	0	7.5	7.5	55	54	1.5

venturi meters out of adjustment part of November. ^d Trial chlorination and trial flocculation. ^e Trial chlorination. actually reduced to 60.6% and 68.6%, respectively, because of lighter solids returned to the river; this reduced percentage respectively, because of lighter solids returned to the river

Screen and Grit Removal.—Sewage was first taken through the screen and grit chambers on June 1, 1938. From that time this unit of the plant functioned in a satisfactory manner. Until May 31, 1940, a total of 79,944 cu ft of screenings and 501,278 cu ft of grit was removed. Related to the 66,864.1 million gal of sewage treated, this amounts to an average of 1.2 cu ft per million gal of screenings and 7.5 cu ft per million gal of grit. Based on a study of 117 screen installations throughout the United States, the average quantity of screenings removed by screens with one-inch openings was 3.1 cu ft per million gal. The quantity of screenings produced by the Twin City plant fortunately was only about one third of the quantity that might have been expected. The quantity of grit removed compares with an average in a num-

ber of plants in the United States of 4.1 cu ft per million gal. A relatively clean grit containing an average of 11.7% of volatile solids was produced.

Velocities in the grit chambers were originally maintained at 0.75 ft per sec. In 1940 velocities during dry weather flows were gradually increased to 1.20 ft per sec for the definite purpose of securing a greater percentage of inert material in the sludge, thereby increasing the solids content of the sludge and reducing the quantity of conditioning chemicals required.

Because of the small quantity of screenings in proportion to grit, it was possible to bury the screenings with an appropriate depth of the relatively clean grit. Shredders were used only to test out the equipment and periodically to keep the equipment in satisfactory operating condition. A variety of material was removed on the hand-raked coarse bar racks (6-in. openings) at the entrance to the plant. Material removed at this point has included both vegetable and animal matter and has ranged from logs and timbers, a section of boiler plate 8 ft by 4 ft, and whole trees to dead dogs and quantities of cattle paunches.

A number of improvement changes were made in this part of the plant which resulted in better and more simplified operation. Some difficulty with wear because of sand entering the lower and intermediate bearings on the screw conveyer grit washers was experienced; but this was partly corrected during 1939 by the manufacturers at no expense to the Sanitary District. However, more positive methods of reducing wear still further have been installed by the Sanitary District, consisting essentially of sealed water-washed and lubricated bearings. After almost one year of operation several bearings of this type installed for observation showed little, if any, wear, and as a result of this experience all bearings have been modified to the new type. From a very troublesome installation requiring replacement of the former type bearings after only one month of operation, this equipment has been converted into a very smooth and satisfactory operating installation. Another item of equipment with which considerable difficulty was experienced in the early part of 1939 was the grit bucket elevator. With grit of certain characteristics, principally that containing high moisture content and a large percentage of "fines," the elevator would not unload satisfactorily, and material would enter the bearings and accumulate on the pulleys and otherwise cause difficulty. By the installation of an air jet to sweep the material out of the buckets, and by installing sealed bearings of a special type, the difficulties with this equipment have been largely overcome.

Sedimentation.—From the time the settling tanks were placed in operation on June 9, 1938, they have operated continuously and satisfactorily. The removals accomplished by the settling tanks were unusually high, from the standpoint of the removals expected, and compared with the removals by most of the other primary settling tanks throughout the United States. The fact that the tanks operated satisfactorily, both from the viewpoint of reliability of equipment and efficiency of removals, is gratifying, considering the fact that the long tanks represent, to some degree, departure from precedent, both for the purpose of effecting construction economies and of accomplishing the higher removals expected from a consideration of a number of theoretical factors.

An effort was made to curtail the removals by the settling tanks, until the South St. Paul and Newport plants were in operation, to an average of approximately 60% removal of suspended solids, by reducing the detention period. With an average detention period for the two years of only 1.5 hr, as compared with a possible detention period of somewhat more than three hours at present flows, the average removal of suspended solids was 69% and of 5-day B.O.D. 38%. Considering the four months of 1939 and 1940, when detention periods of more than 2 hr were maintained (averaging 2.4 hr), the removal of suspended solids averaged 74.4% and of B.O.D. 42.9%. By using all of the settling tanks, a detention period of 3.2 hr can be obtained at present flows, which should result in still higher removals.

Because of the long tanks, the settling tank equipment is operated continuously. Sludge is pumped once each shift, and a determined effort is made to secure as concentrated a sludge as possible. That such efforts were successful is shown by the fact that for the 2-yr period the solids concentration in the raw sludge pumped from the settling tanks averaged 7.7% (92.3% moisture). The skimmings removed from the settling tanks are ejected to an area south of the plant where they are covered with incinerator ash without nuisance, by the simple device of discharging the two materials at the same point.

The aforementioned removals may be compared with expected removals (for a detention period of approximately 2.5 hr) of 56% on a suspended-solids basis and 36% on a 5-day B.O.D. basis. These removals were based on a study of thirty sedimentation plants throughout the United States, reported³ in 1933. It has been suggested that the relatively high removals were due to a relatively low percentage of volatile matter in the raw sewage, but this opinion was determined to be without foundation. As an indication of the fact that the volatiles in the raw sewage are in line with what is experienced elsewhere, the average for the year 1939 was 71.1%.

A number of improvements were made in the appurtenances to the settling tanks to increase dependability, to reduce costs, and to simplify operation. Perhaps the most important is the method of handling scum after its removal from the settling tanks. Automatic skimming mechanisms remove the material from the settling tanks and deposit it in scum troughs from which, with some flushing, it is transported to scum manholes. In the original installation, scum was discharged from this manhole to a sump from which the suction of the scum pumps was drawn. Difficulties were encountered early because of segregation of materials in this sump, and in an effort to remedy this condition the suction was corrected directly to the pumps and various methods of agitation were installed. However, suction and discharge lines, as well as the pumps themselves, frequently became plugged, and in addition to using large quantities of transporting water it was necessary to screen out all material retained on $\frac{3}{4}$ -in. racks. Nevertheless, after four months of operation, the cross-sectional area of the scum discharge line had been reduced to one tenth of its original area by grease incrustations. A number of different methods of handling were considered, from which an ejection method was selected, chiefly because of its possibilities of handling relatively dry scum, which is an important

³ "Factors Affecting the Efficiency of Sewage Sedimentation," by George J. Schroeffer, *Sewage Works Journal*, Vol. 5, 1933, No. 2, pp. 209-232.

consideration in connection with final disposal. This system was installed with the result that dry material could be handled without the necessity of previously removing large pieces of material, and without attendant stoppage, all of which greatly simplified disposal problems. Although this problem may appear unimportant from the standpoint of general plant performance, it has greatly reduced the labor required in this part of the treatment process.

No difficulties were experienced with freezing of sewage in the settling tanks or effluent filters during the winter months. Even with temperatures as low as 25° F below zero, sewage temperatures dropped only about 4° F through the plant. Some difficulty was experienced, however, with the automatic skimming mechanisms during cold weather. Heavy frost gathered on moving parts of the equipment during the period that the mechanism was stationary, with the result that some attachment links were broken. It was found much more economical under these conditions to skim the material into the scum troughs by hand than to attempt to keep the equipment operating.

It was expected at the time of design that sludge as removed from the settling tanks usually would contain from 3% to 6% solids. By careful control of sludge pumping, it has been possible to remove sludge from the tanks averaging between 7% and 8%. In order to handle heavy sludge, containing at times as high as 20% solids, without frequent stoppages, it was found necessary to make provisions for blowing back suction lines with plant effluent or compressed air, on a routine schedule. In this manner, difficulties with sludge suction lines as long as 300 ft were eliminated. Careful control of pumping is required to secure a concentrated sludge. The tendency generally is to pump for too long a period. Except under conditions of storm flow, the usual practice is to operate the sludge pumps three times daily, continuing pumping as long as the sludge remains heavy. As a means of maintaining as concentrated a sludge as possible, specific gravities are determined by weighing calibrated containers of sludge periodically during the pumping period.

One of the problems in the early operation of the plant was the difficulty created by mud and silt that were carried into settling tanks during thaws and storms. This material, which was too fine to be removed by the grit chambers, collected in large quantities in the settling tanks and on several occasions stalled the collector mechanisms, with the result that the tanks had to be pumped down and flushed out with a hose. The difficulties resulted partly from the inability to move the mud and silt through the sludge suction pipes, due to stoppage in the lines, which, even after they were cleaned with compressed air, removed only water. Climatic conditions require the use of large quantities of sand and cinders on the streets during the winter months, amounting to a total of approximately 20,000 cu yd during the winter of 1938-1939. Thaws and early rains carry in a fair percentage of this material. Samples taken at half-hour intervals before, during, and after rains in the early summer indicated suspended-solids strength up to 6,000 ppm, compared with averages of approximately 275 ppm. The percentage of volatile material drops very low during these times. Depending on the intensity and interval between rains, present indications are that flows of more than 250% of the average usually result in settling tank and sludge pumping difficulties. It was found that, when a heavy blanket of normal sewage sludge is carried in the tanks previous to storm flows,

more inert material can be removed effectively from the tanks because of the evident loosening of the mud and silt which ordinarily packs and results in difficult removal problems. As an indication of practice and experience elsewhere, a study of twenty five of the larger plants in the United States showed that the average of the peak flows treated for these plants was 180% of the average flow treated.

It was found desirable to replace the pistons and the collars on the sludge pumps with heavier parts made in the shop of the Sanitary District at a fraction of the cost of such parts from the manufacturers. Although no unusual difficulty was experienced with the lighter pistons, it was felt that it would be economical to install heavier pistons that would permit resurfacing from time to time, thereby prolonging the life. The only difficulty of any type with the sludge collection equipment in the settling tanks, and that of a minor nature, was the breakage of some of the links of the drive chains, especially during the winter months. A completely satisfactory explanation of the cause of this difficulty has not been found as yet, but after experimenting with different materials and types of chains the conclusion has been reached that the installation of chain tighteners will largely eliminate the previous difficulties. Data on removals by sedimentation for the various periods are presented in Table 2.

TABLE 2.—DATA ON REMOVALS BY SEDIMENTATION

Period of observation	Detention period, in hours	RAW SEWAGE STRENGTH (PPM)		PERCENTAGE REMOVAL			Percentage of total solids in raw sludge
		5-Day B.O.D.	Suspended solids	5-Day B.O.D.	Suspended solids	Settleable solids	
July, 1938, to December, 1938, inclusive.....	2.15	200	240	36.0	68.8	90.0	7.15
1939.....	1.3	205	285	38.9	68.9	84.9	7.79
January, 1940, to May, 1940, inclusive.....	1.5	225	300	38.0	70.1	90.8	8.09

Because of early difficulties in placing the plant in operation, the period from July to December, 1938, is not truly representative of normal operation. Considering the latter two periods, the removals of suspended solids averaged 69.5% and of B.O.D. 38.4%, with a detention period of 1.4 hr (less than one half of the possible period with all tanks in service). The increase in the solids content of the raw sludge with experience is shown in Table 2.

Effluent Filters.—The downward-flow magnetite filters were first placed in operation on July 6, 1938. This equipment was purchased under guarantees providing for removals of suspended solids, as follows:

Influent strength (ppm)	Reduction through filters (ppm)
30.....	2
50.....	9
70.....	18
90.....	29
110.....	40
130.....	50

The filters were designed for an average rate of 3 gal per sq ft per min and a short-time maximum rate of 6 gal. This compares with average rates at other plants generally between 2 and 2.5 gal per sq ft per min.

Early operation indicated that the filters would meet the guaranteed removals, but difficulties with sand erosion and displacement occurred at flows greater than the average. The earlier difficulties were felt to be associated with the hydraulics of the structure, considerable erosion near the inlet ports of the beds being experienced. As a means of overcoming this difficulty, the contractor enlarged the influent ports, which apparently solved the erosion problem. However, with this solved, another problem appeared: The sand was displaced, laterally and longitudinally, by the action of the cleaner carriages in passing over the beds. It is a pleasure to report that, as a result of continued efforts over a long period of time, the manufacturer has apparently corrected these difficulties.

The removals of suspended solids by the effluent filters during thirty-four days of intermittent operation, between May 1 and December 28, 1939, have been as follows:

Suspended Solids (ppm):	
Raw sewage.....	280
Filter influent.....	78
Filter effluent.....	56
Percentage of Removal by:	
Settling tanks.....	71.1
Effluent filters.....	8.2
Total.....	79.3
Detention period in settling tanks, in hr.....	1.6
Filter rate, in gal per sq ft per min.....	2.5

During the foregoing period, settling only was used prior to filtration (that is, there was no flocculation or chemical treatment). The sedimentation period prior to filtration was only about one half of the normal detention period in the tanks. Somewhat higher removals prior to filtration can be effected, especially if flocculation (without chemicals) is practiced prior to sedimentation. Partly because of these difficulties, as well as the fact that there operation was not required in any event, the filters were operated only a relatively few days during the 2-yr period covered by this paper.

• SHORT-TIME TESTS OF VARIOUS OPERATIONS

Flocculation Without Chemicals.—During the first two years of operation, treatment consisted essentially of plain sedimentation, but flocculation without chemicals, effluent filtration, chemical treatment, and chlorination were practiced for varying periods of time for test purposes. During June, 1939, flocculation without chemicals was practiced in one battery, and plain sedimentation was continued in the other battery for comparison. An average of approximately 0.035 cu ft of air per gallon was used for flocculation. The

average removals in suspended solids and 5-day B.O.D. were as follows:

Description	Flocculated and settled	Settled only
Suspended solids.....	80.1	75.5
5-day B.O.D.....	49.8	44.2
Period of flocculation, in min.....	32.0	...
Period of sedimentation, in hr.....	1.70	1.64

Chemical Treatment Trials.—During 1939, chemical treatment was not practiced; nor was it considered necessary except to test the various equipment for four days. During this short test period it was impossible to adjust flocculating conditions to secure optimum removals. Ferric sulfate was used as a coagulant and no attempt at *pH*-control was tried. For this test, again, the plant was divided into two halves, in one of which sedimentation alone was practiced; in the other, chemical treatment was used. The results of this short test are as follows:

Description	Plain sedimentation	Chemical treatment
Detention Period:		
Flocculation tanks, in min.....	...	34.3
Sedimentation tanks, in hr.....	1.4	1.85
Air, in cu ft per gal.....	...	0.025
<i>pH</i> -value, raw sewage.....	7.4	7.4
Chemicals added (in ppm of iron)....	...	12
Percentage of Removal:		
Suspended solids.....	73.0	83.4
5-day B.O.D.....	35.3	55.4

Neither of the foregoing tests include effluent filtration, the inclusion of which would increase the removals accomplished.

Disinfection Trials.—Sewage was disinfected with chlorine in half the plant for eleven days in June and July for the purpose of running operating tests on the equipment and obtaining some information on effective dosages. One 16-ton tank car of chlorine was used, and chlorine dosages of 4, 6, and 8 ppm were tried. Pre-chlorination post-chlorination, and split chlorination were used—that is, trials were made applying the chlorine (a) to the raw sewage

TABLE 3.—DISINFECTION OF SEWAGE (AVERAGE VALUES)

Chlorine dosage (ppm)	BACTERIAL COUNT IN RAW SEWAGE (COLONIES PER ML, 48 Hr; AGAR, 37° C)		COLIFORM ORGANISMS IN RAW SEWAGE (<i>Coli-Aerogenes</i> PER ML; MOST PROBABLE NUMBER)	
	Total	Percentage reduction	Total	Percentage reduction
4	9,200,000	67.1	170,000	56.1
6	7,600,000	97.9	230,000	97.4
8	6,200,000	99.64	200,000	99.95

only (pre-chlorination), (b) to the settled effluent only (post-chlorination), and (c) dividing the dosage between the raw sewage and the effluent (split chlorination). The flocculating tanks were used to provide approximately 30 min contact time for the chlorine added to the settling tank effluent. To avoid

any possible changes in bacterial counts in stored samples, only grab samples collected during the morning and afternoon were used to measure the bacterial kill or disinfection. A summary of the results obtained is given in Table 3.

The results of pre-chlorination, post-chlorination, and split chlorination are not shown separately in Table 3. Although they were quite uniform, the tests covered too short a time to justify any definite conclusion regarding the relative effectiveness of the various points of chlorine application. The results indicate that a satisfactory kill of bacteria can be obtained with a comparatively small chlorine dosage of 6 to 8 ppm.

Recirculation of a Percentage of Incinerator Ash.—During the latter part of September, 1940, the return of a percentage of incinerator ash to the raw sewage ahead of the screen and grit chambers was begun. This experiment was conducted as a result of laboratory experiments made approximately two years previously in an effort to improve vacuum filtration and reduce the quantity of conditioning chemicals required. Because this test was begun only recently, it is too early to present conclusive data, but present indications are that very satisfactory reductions in chemicals required for sludge conditioning have been effected, in addition to which appreciable increase in the efficiency of sedimentation resulted. It is the expectation that flocculation will further increase the removals accomplished.

SLUDGE DISPOSAL

Sludge Dewatering; Character of Sludge.—The raw sludge delivered to the concentration tanks varies over quite a wide range in quantity and in the characteristics that affect filtration and incineration. Referring to Table 4, the raw sludge as removed from the settling tanks in the period from September 1, 1938, to May 31, 1940, averaged as follows: *pH*, 6.0; moisture content, 92.26%; and volatile solids, 64.8%. After concentration for a period which generally varies from one to two days, the *pH*-value of the sludge had changed to 5.9, the moisture content to 90.88%, and the volatile solids to 63.6%.

Sludge Conditioning.—The chemicals used for conditioning, except for short periods when ferric chloride alone was used as an experiment, have been lime and ferric chloride. During the period covered by this paper, the quantity of ferric chloride has averaged 2.18% of the weight of dry sewage solids, and the lime 4.87%. However, during the last six months of this period, the quantity of chemicals averaged somewhat less, or 1.89% ferric chloride and 4.37% lime, as compared with the requirements of 2.68% and 9.74%, respectively, during the first six months of operation. Control of the quantity of chemicals used for sludge conditioning has been by a physical examination of the cake and the sludge by the operators, checked by relatively frequent laboratory tests. Early in 1940 continuous control of filtration by the filter operators was begun by means of Bueckner funnel tests. It is of interest to mention that the period of sludge conditioning has been gradually shortened so that in 1940 it averages about 6 min.

Proper mixing and agitation of conditioned sludge prior to filtration is of utmost importance, especially with heavy sludge. The use of large quantities of low-pressure air administered at a number of points resulted in improvement

in filtration and a consequent reduction in chemicals required. As originally installed, high-pressure air was supplied through porous tubes. Upon failure of this arrangement to perform in a completely satisfactory manner, various methods were tried, culminating in one in which a total of eighteen small pipes

TABLE 4.—SUMMARY OF DATA ON SLUDGE CHARACTERISTICS
AND VACUUM FILTRATION
(September, 1938, to May, 1940, Inclusive)

Month	pH-VALUE		TOTAL SOLIDS (%)			VOLATILE SOLIDS (%)			AVERAGE TONS OF MATERIAL DAILY			CONDI- TIONING CHEMICALS, % OF SEWAGE SOLIDS		Average rate, in lb per sq ft per hr
	Raw sludge	Thick- ened sludge	Raw sludge	Thick- ened sludge	Filter cake	Raw sludge	Thick- ened sludge	Filter cake	Dry sewage solids filtered	Wet cake pro- duced	In- ciner- ator ash pro- duced	Ca O	Fe Cl ₂	
1938:														
Sept. . . .	6.4	5.9	7.80	9.10	34.5	59.3	59.5	51.4	44.5	136.7	23.0	10.30	3.30	7.31
Oct. . . .	6.3	6.3	6.29	8.20	34.3	65.0	64.2	52.6	60.8	194.0	31.5	11.70	2.69	5.68
Nov. . . .	5.9	6.1	7.40	9.16	35.1	63.5	62.7	52.6	75.0	216.0	36.0	10.70	2.58	5.02
Dec. . . .	5.8	5.8	7.10	8.65	34.2	67.9	66.2	54.7	97.6	317.0	49.0	11.20	2.80	5.05
1939:														
Jan. . . .	6.2	5.9	7.70	9.52	36.1	64.6	63.9	55.9	101.6	326.6	52.1	8.78	2.36	6.21
Feb. . . .	6.1	6.0	7.18	8.87	34.8	67.8	66.8	62.3	104.0	309.4	41.0	5.76	2.34	4.47
Mar. . . .	6.1	6.0	10.25	11.18	39.1	56.6	55.7	54.3	121.1	285.5	50.0	3.24	1.76	4.90
Apr. . . .	6.0	5.9	8.04	9.61	35.0	65.2	63.2	59.1	102.2	313.3	46.7	4.80	2.06	4.84
May . . .	5.8	5.7	9.08	10.06	35.3	64.8	64.1	60.1	107.7	320.4	48.7	4.72	2.02	5.29
June . . .	5.8	5.7	8.53	10.74	38.3	59.2	56.5	53.9	124.2	317.6	57.2	4.61	2.07	5.29
July . . .	5.7	5.6	7.58	9.05	35.3	62.9	60.8	56.6	90.9	273.5	42.6	5.83	2.44	4.97
Aug. . . .	5.7	5.6	7.22	8.42	33.9	64.7	63.1	58.2	83.7	269.3	39.0	5.97	2.14	4.71
Sept. . . .	5.8	5.7	6.63	8.22	33.0	66.4	64.7	60.6	97.7	307.5	41.4	5.63	2.11	4.63
Oct. . . .	5.8	5.8	6.64	7.85	32.7	69.3	67.3	62.9	99.9	320.2	38.9	5.82	1.89	4.32
Nov. . . .	5.7	5.7	7.29	7.82	31.6	71.9	71.1	65.5	97.9	305.4	33.4	6.71	1.92	3.73
Dec. . . .	5.7	5.8	7.33	8.18	31.7	72.4	71.8	66.8	89.1	290.4	30.5	6.26	2.06	3.99
1940:														
Jan. . . .	5.8	5.8	7.21	8.05	31.9	72.3	72.2	68.6	86.1	264.7	26.5	4.65	2.17	3.80
Feb. . . .	5.9	5.8	7.87	8.81	33.7	67.7	67.3	64.4	110.1	337.0	40.4	4.81	1.90	4.35
Mar. . . .	6.2	6.0	8.77	10.3	38.1	57.0	56.4	54.6	131.8	340.2	56.7	3.79	1.63	4.88
Apr. . . .	6.2	6.1	8.69	10.4	37.8	58.6	56.6	54.6	119.9	300.1	51.3	3.53	1.70	4.61
May . . .	6.1	6.0	7.93	9.43	36.2	63.1	61.4	58.6	121.7	324.3	47.4	3.70	1.88	4.27
Average	6.0	5.9	7.74	9.12	34.9	64.8	63.6	58.5	98.5	289.0	42.1	6.29	2.18	4.87

was added to each tank and excess low-pressure air from the blowers, provided for removing cake from the filters, was utilized for agitation. Means for distributing the chemicals more uniformly through the tank were provided, and experiments were made to determine the proper sequence of chemical additions. Studies are under way to effect additional improvements in sludge conditioning.

It has been found that the quantity of chemicals required for sludge is less with heavy sludge, which condition is usually associated with rains carrying inert material into the sewage. With the particular design of sludge piping installed in this plant, sludge containing more than 10% to 12% solids usually requires thinning with plant effluent to permit its movement through the piping system.

Vacuum Filtration.—Dewatering of sludge was begun on July 2, 1938. The sludge cake produced averaged 34.9% solids (65.1% moisture) in the 2-yr

period. As a monthly average, during March, 1939, the moisture content was as low as 60.9%. Daily averages have been lower than 50%, these low moisture contents being associated with rains and periods when the volatile solids were exceptionally low. The volatile solids in the filter cake vary widely. During the period from September, 1939, to May, 1940, the volatile solids in the filter cake averaged 58.5% with a range in monthly averages from 51.4% to 68.6%. Daily variations are still greater. In the period since operation started to May 31, 1940, 183,497.6 tons of filter cake were produced by the plant. For the period from September 1, 1938, to May 31, 1940, the quantity of dry sewage solids has averaged 98.5 tons daily, equivalent to 289.0 tons of filter cake daily.

As in sludge conditioning, a number of improvement changes were made in this part of the plant which have resulted in simplified operation and marked reductions in operating cost. As an example of the effect of these changes, including those in sludge conditioning, during early operation approximately 3% ferric chloride and 10% lime were required, which has been reduced by various means to somewhat less than 2% and 5%, respectively. On the basis of 100 tons of dry sewage solids daily, and chemical prices current at this time, this amounted to a saving of \$100 per day in chemical costs.

The reduction in the use of lime resulted in other operating advantages. In early operation, difficulty was experienced with lime-blinding of the cloths, and after a period of several months' operation the openings on the screen supporting the cloth had been reduced to less than one quarter of their original area by calcium carbonate deposits. A number of methods of removing this material were tried. At one time it appeared desirable to purchase new screens at a cost of \$193 per filter. Finally the screens were removed and sandblasted at a cost of only \$15 per filter, with entirely satisfactory results. With the reduction in the quantity of lime, the necessity of cleaning screens has been practically eliminated.

To remove calcium carbonate from the screens, conditioning with ferric chloride alone was tried. It was determined, after a short time of operation on two of the filters, that this method, although effective, was not economical. However, the experiment permitted a determination of the effectiveness of this chemical alone for conditioning. The rate of filtration as compared with lime and ferric chloride was reduced, ranging from 3 to 4 lb per sq ft per hr for dosages of 3% to 4% of ferric chloride.

For the entire period the average filter rate was 4.87 lb per sq ft per hr. During the early period when higher chemical dosages were used, the rates were considerably higher, averaging as much as 10 lb for various periods of time. In addition to the higher operating costs to secure greater rates, the production of a thick cake presented certain undesirable operating problems in the conveying of the cake. In view of the fact that the increased power cost by using more filters was insignificant, and that the extra cost of cloths was small in comparison with the savings in chemicals which could be effected, the desirability of operating with lower filter yields was apparent. In emergencies, or under extreme peak conditions, it is necessary to use higher chemical dosages to increase the filter rate.

The problem of increasing filter cloth life has been given due consideration, brushing and washing with steam and air being tried progressively. In the early period of operation, cloths generally had to be removed after about 150 hours of use. The need for removal was dictated by the fact that lime-blinding necessitated higher chemical dosages and more frequent washing, and ripping or rotting at seams required the use of a large number of tin patches (as many as 30 to 40 on a filter) even with this short life. With reduced lime dosage the economical cloth life has been increased to somewhat more than 300 hours (periods of usage of 500 to 600 hours were common for several months). Beyond this time increased chemical feed is required, incommensurate with the cost of cloth replacement, especially in view of the reduction in cost of cloth which has been effected. The ability to purchase identical cloths locally at a fraction of their original cost has made it possible to change filter cloths more frequently than was formerly the case. In addition, the new cloths are sewed with a different type of thread and with a lock stitch so that difficulties at the seams, requiring patching, have been eliminated.

Changes have been made in the location and angle of incidence of water sprays used in washing filters as compared with the type ordinarily manufactured. Various types of sprays arranged at different angles relative to the drum surface were tried. The original sprays were reversed, and located so that water would impinge almost at right angles to the cloth on the drum. The new arrangement permitted better washing in one third of the time previously required. Less water is used, and the necessity of brushing the cloth has been eliminated, thus insuring a longer cloth life.

The problem of conveying filter cake, containing varying quantities of water, on belts presents many difficulties that are sometimes not recognized. Stoppages are quite frequent with relatively minor restrictions in openings around and over belts and at transfer points. These problems are aggravated with thick cake (heavier than $\frac{3}{8}$ in.). It was found early that large blocks of cake, as long as 8 ft and as wide as the filter drum, were broken off as the drum revolved. Various methods of breaking up this cake were considered, resulting in the selection of the simple and effective expedient of installing sloping $\frac{1}{2}$ -in. rods on 12-in. centers in the filter take-off plates. This device cuts the cake into ribbons one foot wide, which break off irregularly and eliminate difficulties from this cause.

Incinerators.—Incineration of sludge was begun on July 21, 1938, and 24-hr operation on July 29. On August 4, 1938, the first incinerator placed in operation became heated to the point at which the manufacturers' representatives considered that it was necessary to shut down in order to insulate the rabble arms on certain of the hearths. Incinerator temperatures up to 2,200° F were recorded as compared with expected maximum temperatures of 1,600° F. The high temperatures required the replacement of several of the rabble arms and a section of the central shaft on this incinerator.

The explanation for this condition is simple. Instead of requiring approximately \$2,500. worth of fuel oil a month, as estimated, it was found early that under practically all conditions of operation an excess of heat existed, and no fuel was required for actual incineration. It was the manufacturers' expecta-

tion that the location of burning, and the temperatures of combustion, would be controlled by the use of varying quantities of oil introduced at different points. Instead, it was necessary to dissipate heat to the extent of more than 6,000,000 Btu per hr per incinerator. In the beginning the manufacturers attempted to accomplish this by introducing water into the furnace through sprays. Although this was effective, it was considered unsatisfactory by the Sanitary District to use the large quantity of water required. After experimenting with different methods for solving their difficulties, the manufacturers installed a by-pass to conduct preheated air direct to the stack. In this way air at room temperature could be used for combustion, and the water would be reduced to reasonable quantities and its addition required on less frequent occasions.

Continuous operation of all the incinerators was possible beginning November 17, 1938, and in accordance with the specifications the acceptance tests were begun thirty days later, on December 17, 1938, and completed satisfactorily on March 17, 1939.

The method of correcting some of the early difficulties with overheating, etc., was a matter of thorough investigation over a period of five months, and presented some rather interesting problems. Progressively, a number of solutions were considered, and the methods finally adopted were those selected after careful consideration of all factors by the Sanitary District and the contractor. Some of the difficulties appeared insuperable at times, but it is a pleasure to report that they were successfully overcome by the contractor, at no expense to the Sanitary District.

From July 21, 1938, to May 31, 1940, 182,097.6 tons of filter cake were incinerated. The average Btu per pound of combustible solids in the sludge during this period was 10,800, and the average volatile solids in the sludge cake during the period covered herein amounted to 58.5%. The sludge cake was reduced by incineration to an ash containing less than 2% volatile matter (actual average 1.6% during guarantee period). As shown in Table 4, the moisture content in the sludge cake as produced by the filters has averaged 65.1% during the period from December, 1938, to May, 1940, inclusive.

The comparison between the quantity of material incinerated and the guaranteed capacity of the incinerators is interesting. Over a 24-hr period, a maximum of 523.8 tons of filter cake or a total of approximately 225 tons of dry solids have been incinerated. The capacity of each of the three incinerators is 60 tons of dry solids per twenty-four hours. As much as 78.5 tons of dry solids have been incinerated in one incinerator in a 24-hr period. Hourly rates considerably in excess of the foregoing have been handled successfully. For the period from December 1, 1938, to May 31, 1940, the number of tons of dry solids incinerated per day in each incinerator has averaged 55.1 tons as compared with the maximum guaranteed capacity of 60 tons daily.

During the period from December, 1938, to May, 1940, inclusive (see Table 5), 123,911 gal of fuel oil were required by the incinerators. Practically all of this was required in bringing furnaces up to temperatures from a cold condition or in cooling them down, and in holding temperatures in furnaces out of service. The aforementioned quantity of oil amounts to an average of 2.3 gal per ton of dry solids incinerated. Power required for incineration amounted to an average of 17.3 kw per hr per ton of dry solids.

Miscellaneous Sludge Disposal.—Investigations are in progress in connection with the utilization of sludge and ash for various purposes. A research fellowship has been established with the Agricultural College of the University of Minnesota, at Minneapolis, to explore the fertilizing or soil conditioning

TABLE 5.—DATA ON SLUDGE INCINERATION,
DECEMBER, 1938, TO MAY, 1940

Month	TONS OF SLUDGE INCINERATED		Dry tons per incinerator day	Oil used (gal)	Power (kw-hr)	Month	TONS OF SLUDGE INCINERATED		Dry tons per incinerator day	Oil used (gal)	Power (kw-hr)
	Cake basis	Dry basis					Cake basis	Dry basis			
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
1938:						1939:					
Dec.	9,826.4	3,346.5	53.8	7,573	58,603	Oct...	9,284.5	3,036.4	51.2	6,629	54,761
1939:						Nov...	7,023.4	2,222.3	45.0	6,836	45,257
Jan...	10,119.0	3,654.5	60.4	4,195	54,782	Dec...	9,002.6	2,850.9	46.5	3,557	53,822
Feb.	8,663.2	3,022.3	56.6	6,069	50,943	1940:					
Mar.	8,851.9	3,349.5	61.3	6,793	55,829	Jan...	8,206.1	2,618.0	43.5	2,767	52,509
Apr.	9,398.7	3,321.6	61.4	5,634	54,205	Feb...	9,773.0	3,295.2	51.4	9,776	61,504
May	9,932.2	3,579.9	61.5	10,470	61,569	Mar...	10,177.2	3,874.7	58.5	10,938	66,098
June	9,528.9	3,649.1	64.0	6,188	55,572	Apr...	8,964.5	3,391.4	55.7	6,604	56,604
July...	8,479.3	3,007.7	57.5	10,763	52,061	May...	9,844.0	3,564.6	53.3	8,078	59,278
Aug.	8,348.5	2,847.6	55.0	14,064	52,069	Total...	164,646.9	57,793.6	...	132,911	997,969
Sept.	9,223.5	3,071.4	55.0	5,977	52,503	Average	9,147.0	3,211.0	55.1	7,384	55,443

possibilities of sludge cake as produced by the filters and its use after storing for periods of time. Experiments have been made on the production of partly dried material by removal of a portion of the sludge from the top hearths of the incinerator, using the heat of the remainder of the sludge incinerated for drying purposes. An unusual degree of interest has been shown in the use of the material by farmers, nurseries, golf courses, parks, etc., but to 1940 no material was distributed except to those working in cooperation with the Agricultural College. This has been done with the express purpose of determining the advantages and disadvantages of the material, as well as the proper conditions of use, before general distribution and sale. From the interest and requests to date, it appears likely that a relatively large proportion of the material may be disposed of in this manner during a part of the year.

The use of ash has been investigated. Some possible uses that have been suggested are: A binder or filler for various products, a soil conditioner, an ingredient for certain kinds of cement, etc. About 400 tons of the material have been disposed of as filler, in lieu of pulverized limestone, for a commercial fertilizer. A typical analysis of the ash from this plant is as follows:

Silica as Si O ₂	43.6%
Calcium as Ca O	26.3%
Aluminum as Al ₂ O ₃	9.9%
Iron as Fe ₂ O ₃	6.9%
Potassium as K ₂ O	2.1%
Phosphorus as P ₂ O ₅	0.7%
Miscellaneous	10.5%

Another interesting possibility relates to the utilization of the heat resulting from incineration. Various possibilities have been developed which may result in the generation of sufficient steam for all heating purposes and electric power requirements.

Everything considered, the methods of sludge disposal used at this plant have proved quite satisfactory, and a number of possibilities for still further improvement are under consideration. The successful and economical operation has been very gratifying, and at times such operation was an absolute necessity, especially when it is considered that the quantities of material to be handled were considerably greater than originally expected (monthly averages as high as 131.8 tons daily, as compared with expected average quantities of 76.5 tons daily).

COST FEATURES

Construction Cost.—The total construction cost of the plant was approximately \$3,750,000 or \$28,000 per million gal of capacity. The distribution of this cost was about as follows: Screen and grit chambers with chemical tower, \$540,000; settling tanks, effluent filters, and appurtenances, \$1,630,000; filtration and incineration building, \$1,240,000; laboratory and administration building, \$175,000; and miscellaneous, \$165,000. Equipment represents about \$1,500,000 of the total cost of \$3,750,000.

Operation and Maintenance Costs.—The total operation and maintenance costs for the period from June 1, 1938, when preliminary treatment was first begun, to May 31, 1940, was \$587,136.03 (see Table 6(a)). During this period 66,864.1 million gal of sewage was treated, bringing the average cost per million gallons to \$8.80. The entire flow was not received until about July 15, 1938, and furthermore operation during the early months was curtailed because of the manufacturer's difficulties with incinerator operation. For these and other reasons the costs during the early period of operation in 1938 are not typical. During 1938, for example, the average cost per million gallons was \$10.65, which compares with \$8.20 for 1939, and \$8.35 for the first five months of 1940. Of the total cost during the first two years of operation, maintenance costs totaled \$45,884.23, or 7.8% of the total.

Table 6(b) shows the distribution of operation and maintenance costs by departments for the three periods. In order of descending cost importance the following are the most important items: Vacuum filtration, incineration, supervision and engineering, administrative expense, laboratory expense, sedimentation, etc. This order will be varied, of course, whenever chemical treatment and chlorination are practiced.

Although what follows concerning the comparison between actual expenditures and funds available (see budget provisions, Table 6(b)) may not be of general interest, it is nevertheless a matter of local gratification that it has been possible to hold expenditures within budgeted funds. During the first two years of operation, expenditures totaled \$587,136.03 as compared with budget provisions of \$917,164.81. It is a fact that, although the deliberate reduction in treatment provided (until communities and industries immediately downstream provided comparable treatment) resulted in certain savings, a large part of the reduction in cost was possible by the operation economies effected.

It is interesting to note the large proportion of the total cost directly chargeable to sludge disposal. During 1939, for example, \$136,815.43 was expended for vacuum filtration and incineration, which represents \$3.70 per million gal, or approximately 45% of the entire cost of operation and maintenance.

TABLE 6.—COSTS OF SEWAGE TREATMENT AND SLUDGE DISPOSAL IN THE TWO YEARS ENDING MAY 31, 1940

Dept. No.	Description	June 1, 1938 Dec. 31, 1938	Jan. 1, 1939 Dec. 31, 1939	Jan. 1, 1940 May 31, 1940	June 1, 1938 May 31, 1940
(a) TOTAL AND UNIT COSTS ^a					
	Total operation and maintenance ^b ...	157,868.13	301,538.98	127,728.92	587,136.03
	Maintenance only.....	9,564.26	24,012.46	12,307.51	45,884.23
	Sewage Treatment:				
	Mgd.....	(14,826.1)	(36,705.3)	(15,332.7)	(66,864.1)
	Dollars per mgd.....	10.65	8.20	8.35	8.80
	Disposal:				
	Tons of dry sludge solids.....	(10,248.0)	(37,703.0)	(17,100.0)	(65,051.00)
	Dollars per ton of dry solids.....	15.40	7.98	7.45	9.05
	Cost of sludge disposal only.....	69,840.65	136,815.43	54,082.66	260,738.74
	Cost of sludge disposal, per ton of dry solids.....	6.80	3.63	3.15	4.01
(b) DISTRIBUTION BY DEPARTMENTS					
1	Administrative expense.....	13,180.40	27,822.37	11,712.34	52,715.11
2	Supervision and engineering.....	13,044.72	30,268.52	12,306.69	55,619.93
3	Laboratory expense.....	10,908.35	19,715.30	7,759.84	38,383.49
4	Grit and screenings removal.....	6,658.31	11,978.33	3,910.26	22,546.90
5	Grit and screenings disposal.....	7,024.42	10,764.19	3,950.09	21,738.70
6	Chemical handling and feeding (sewage treatment).....	789.76	895.12	76.65	1,761.53
7	Disinfection.....	227.42	1,132.38	3.11	1,362.91
8	Flocculation.....	490.99	1,460.15	177.80	2,128.94
9	Sedimentation.....	12,207.31	18,482.73	6,803.05	37,493.09
10	Scum disposal.....	4,483.56	3,285.05	855.66	8,624.27
11	Effluent filtration.....	1,783.46	2,369.11	1,031.68	5,184.25
12	Effluent water supply.....	1,371.37	3,498.06	905.83	5,775.26
13	Vacuum filtration.....	50,419.48	99,448.39	33,590.09	183,457.96
14	Sludge incineration.....	21,042.50	37,365.04	13,785.79	72,193.33
15	Heating.....	 ^c	9,815.25	5,175.50	14,990.75
16	Shop.....	 ^c	2,862.27	1,098.18	3,960.45
17	Roads, walks, grounds.....	1,848.95	2,485.67	420.85	4,755.47
18	Odor control.....	 ^c	10.00		10.00
19	Interceptor sewers.....	670.37	1,234.32	100.25	2,004.94
20	Miscellaneous and unassigned.....	 ^c	8,292.39	7,786.02	16,078.41
	Total.....	146,151.37	293,184.64	111,449.68	550,785.69
	Miscellaneous income.....		713.40	222.01	935.41
	Cost of operation and maintenance.....	146,151.37	292,471.24	111,227.67	549,850.28
	Increases in assets, materials and supplies inventory, etc.....	11,736.76	9,065.74	16,501.25 ^d	37,303.75
	Grand total.....	157,888.13	301,536.98	127,728.92	587,154.03
	Budget provisions.....	240,097.93	478,066.88	199,000.00	917,164.81

^a All units are dollars except values in parentheses. ^b Including capitalized improvement changes. ^c Distributed in other departments. ^d Includes \$12,307.51 undistributed maintenance costs.

nance. In 1938 the total cost of sludge disposal was \$69,840.65, during which period a total of 10,248 tons of dry sewage solids were processed. The cost of dry sewage solids was approximately \$6.80 per ton. For the more representative period of 1939, during which time reductions in sludge conditioning chemicals were effected, the total cost per ton was \$3.63, which was further

reduced in the first five months of 1940 to \$3.15 per dry ton. Monthly data on the cost of sludge disposal is shown in Table 7.

Of the total cost of disposal for the period from January, 1939, to May, 1940, inclusive (\$191,311.19), filtration cost was \$137,310.48, or 72%, and

TABLE 7.—OPERATION, MAINTENANCE, AND UNIT COSTS OF SLUDGE
FILTRATION AND INCINERATION
(December, 1938, to May, 1940, Inclusive)

Month	OPERATION AND MAINTENANCE COST			TONS SLUDGE		OPERATION AND MAINTENANCE COST PER DRY TON		
	Vacuum filtration	Inciner- ation	Total	Dry sewage solids basis	Cake basis	Vacuum filtration	Inciner- ation	Total
1938:								
Dec.....	\$13,737.75	\$5,680.77	\$19,418.52	3,067	9,826.4	\$4.48	\$1.85	\$6.33
1939:								
Jan.....	10,594.05	2,716.65	13,310.70	3,150	10,119.0	3.37	0.86	4.23
Feb.....	9,334.35	2,977.01	12,311.36	2,920	8,663.2	3.19	1.02	4.21
Mar.....	8,364.06	2,800.75	11,164.81	3,755	8,851.9	2.22	0.75	2.97
Apr.....	8,095.15	2,444.49	10,539.64	3,065	9,368.7	2.64	0.80	3.44
May.....	8,021.87	2,960.57	10,982.44	3,339	9,932.2	2.40	0.89	3.29
June.....	9,178.08	3,179.92	12,358.00	3,727	9,528.9	2.46	0.86	3.32
July.....	7,239.88	3,630.36	10,870.24	2,818	8,479.3	2.57	1.29	3.86
Aug.....	6,481.29	3,329.77	9,811.06	2,593	8,348.5	2.50	1.29	3.79
Sept.....	7,704.60	3,651.00	11,355.60	2,929	9,223.5	2.63	1.25	3.88
Oct.....	8,180.68	3,798.52	11,979.20	2,896	9,284.5	2.82	1.31	4.13
Nov.....	7,844.92	3,257.00	11,101.92	2,251	7,023.4	3.48	1.45	4.93
Dec.....	8,212.56	3,639.68	11,852.24	2,762	9,002.6	2.97	1.32	4.29
1940:								
Jan.....	7,079.14	3,049.43	10,128.57	2,668	8,206.1	2.65	1.14	3.79
Feb.....	6,590.02	2,976.13	9,566.15	3,192	9,773.0	2.07	0.93	3.00
Mar.....	7,521.09	3,760.62	11,281.71	4,087	10,177.2	1.84	0.92	2.76
Apr.....	8,883.73	3,642.30	12,526.03	3,596	8,964.5	2.47	1.01	3.48
May.....	7,344.90	3,235.30	10,580.20	3,772	9,844.0	1.96	0.86	2.82
						Average:		
Total ...	\$150,408.12	\$60,730.27	\$211,138.39	56,587	164,647	\$2.67	\$1.06	\$3.73

TABLE 8.—DISTRIBUTION OF OPERATION AND MAINTENANCE COSTS FOR THE
PERIOD JANUARY 1, 1939, TO MAY 31, 1940

Description	FILTRATION		INCINERATION		TOTAL	
	Cost	%	Cost	%	Cost	%
Operation:						
Salaries and wages.....	33,198.51	24.2	27,217.36	50.4+	60,415.87	31.5
Material and supplies.....	75,603.40	55.0	5,606.26	10.4	81,209.66	42.5
Power and light.....	12,652.80	9.2	13,248.20	24.5	25,901.00	13.6
Miscellaneous.....	4,651.42	3.4	389.08	0.7	5,040.50	2.6
Maintenance.....	11,204.35	8.2	7,539.81	14.0+	18,744.16	9.8
Total.....	137,310.48	100.0	54,000.71	100.0	191,311.19	100.00

incineration \$54,000.71, or 28%, of the total cost. As further shown in Tables 8 and 9, the principal item of cost in connection with sludge dewatering is material and supplies, whereas salaries and wages constitute the principal cost factor in sludge incineration. Maintenance (including wages) amounts to

about 10% of the entire cost of disposal. Although it is true that maintenance costs in the first two years of operation of a plant are expectedly lower than they will be after, say, ten years (and that therefore it might be held to follow that costs will increase after a period of years), operation of this plant to 1940

TABLE 9.—SUMMARY OF UNIT COSTS

Treatment	DRY SEWAGE SOLIDS		FILTER CAKE		Average cost (cost per month)
	Tons	Cost per ton	Tons	Cost per ton	
Filtration.....		\$2.56		\$0.89	\$3,080
Incineration.....		1.01		0.35	3,180
Total.....	53,520	\$3.57	154,820.6	\$1.24	\$11,260

indicates that such apparent increase will be offset by economies effected in other cost items of greater magnitude.

The total operating and maintenance personnel, including administration, supervision, laboratory, etc., numbers 74 employees, involving a monthly payroll of approximately \$13,500. Operation and maintenance costs are financed in the two cities by sewer rental charges which in one city are based on the volume of water consumed, and in the other on the basis of the size of water meter installed on the premises.

MISSISSIPPI RIVER CONDITIONS

General Data.—A very marked improvement in the physical, biochemical, and bacteriological condition of the Mississippi River became apparent shortly after the Twin Cities' sewage treatment plant was placed in operation. With the passage of time and the gradual elimination of "loose ends" of pollution, the river condition through Minneapolis and St. Paul has gradually approached that entering Minneapolis at Camden, Minn. (see Fig. 2).

Through the Twin Cities the floating scum, sleek and large masses of sludge which in former summers covered as much as 50% of the water surface in some areas, has practically disappeared. The foul odors of past years are no longer in evidence. Game fish are returning to this section of the river which previously did not provide satisfactory environment for even the rougher forms of fish life. Boating is being enjoyed by an increasing number of people.

Occasionally some debris is seen floating on certain sections of the river following heavy storms. The Twin Cities' sewer systems are combined and therefore both storm water and sewage must overflow to the river during heavy storms. The sewer departments of both cities strive to hold such overflow to a minimum by the use of automatic regulators and by inspecting the sewer outlets following storms. The Sanitary District also makes a monthly inspection of all former sewer outlets and reports its findings to the sewer departments of both cities.

The river below the Twin Cities has shown an improvement although it is still seriously affected by the untreated sewage and packing plant wastes from South St. Paul and Newport. Some idea of the relative effect of the sewage and

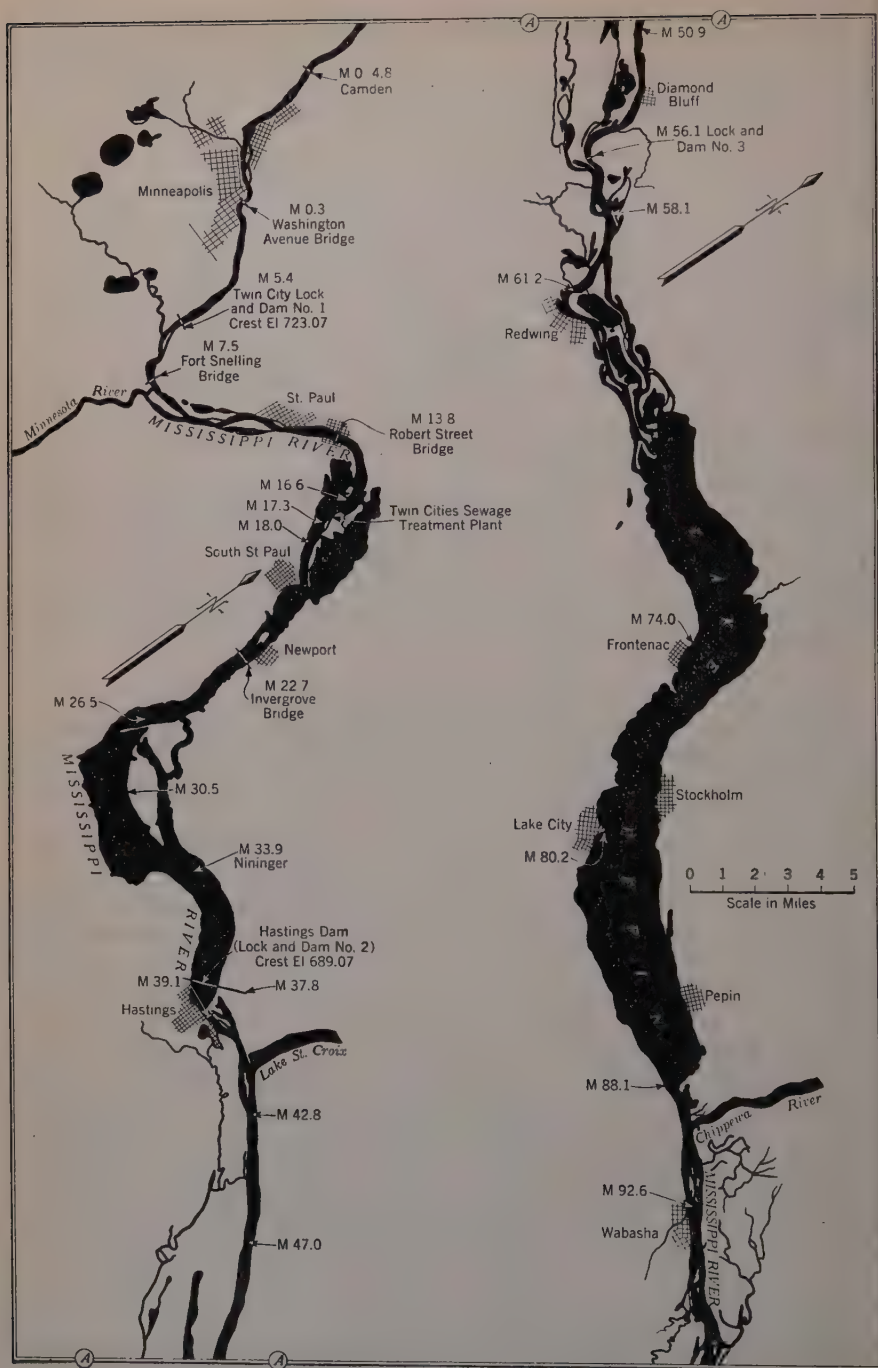


FIG. 2.—LOCATION MAP, MISSISSIPPI RIVER AT MINNEAPOLIS, MINN.

wastes from South St. Paul and Newport can be had from the fact that the population equivalent of the untreated sewage and wastes from these communities varies from 200,000 to 400,000, depending upon the animal kill, as compared with a total population equivalent of untreated sewage from the Twin Cities of from 1,000,000 to 1,200,000. It is unfortunate that at this writing (1940) all sewages and wastes from the metropolitan area are not receiving treatment so as to permit a more definite comparison of river conditions farther downstream "before and after."

Each winter commercial fishermen seine rough fish from the river at Grey Cloud Island, 14 miles below St. Paul. They report the presence of crappies and sunfish for the first time during the winter of 1939, indicating that some of the better forms of fish life are returning even to this section of the river. At other points on the river, both upstream and downstream from the treatment plant, catches of game fish are reported by fishermen, where formerly only the more hardy varieties of "rough fish" were able to survive.

Analytical Data.—The general evidence of improved river conditions discussed herein is substantiated by the analytical data on the river water samples collected regularly for analysis. The continuous record of analytical data on the Mississippi River and its tributaries in the period from 1926 to 1940 is valuable in a comparison of river conditions.

Data contained in Table 10 show the improved condition of the water at the

TABLE 10.—AVERAGE ANALYTICAL DATA AT STATION M 5.4, TWIN CITY LOCK AND DAM, FOR JULY, AUGUST, AND SEPTEMBER

Year	1929	1930	1935	1937	1938	1939
Dissolved oxygen, ppm.....	3.60	3.85	1.70	1.75	6.60	6.60
B.O.D., ppm.....	4.40	4.65	3.85	4.25	1.80 ^a	1.85 ^a
Total bacterial count per ml.....	480,000	237,000	400,000	375,000	17,000 ^a	11,000 ^a
Coliform organisms per ml.....	3,900	8,900	3,100	2,800	325 ^a	110 ^a
River flow at Twin City Lock and Dam, cu ft per sec.....	4,190	3,680	2,885	2,845	4,525	3,274

^a At station M 7.5, Fort Snelling Bridge.

Twin City Lock and Dam during the summers of 1938 and 1939 compared with previous summers before the discharge of sewage into the pool had been stopped. The years 1929, 1930, 1935, and 1937 were selected for comparison because the river flows for those summers most nearly correspond with the flows for the summers of 1938 and 1939. As indicated in Table 10, the dissolved oxygen in the river passing the Twin City Lock and Dam has materially increased, whereas the biochemical oxygen demand and bacterial counts have greatly decreased since the Twin Cities' sewage disposal system was placed in operation in the summer of 1938. This improvement is shown further in Fig. 3, containing a plot of the river data at this station since 1926.

That the river condition through the Twin Cities is gradually approaching that entering Minneapolis is shown by the data contained in Table 11 (see also Table 10). Some added pollution is indicated, however, probably made up from the Minnesota River pollution, from storm water overflow through the

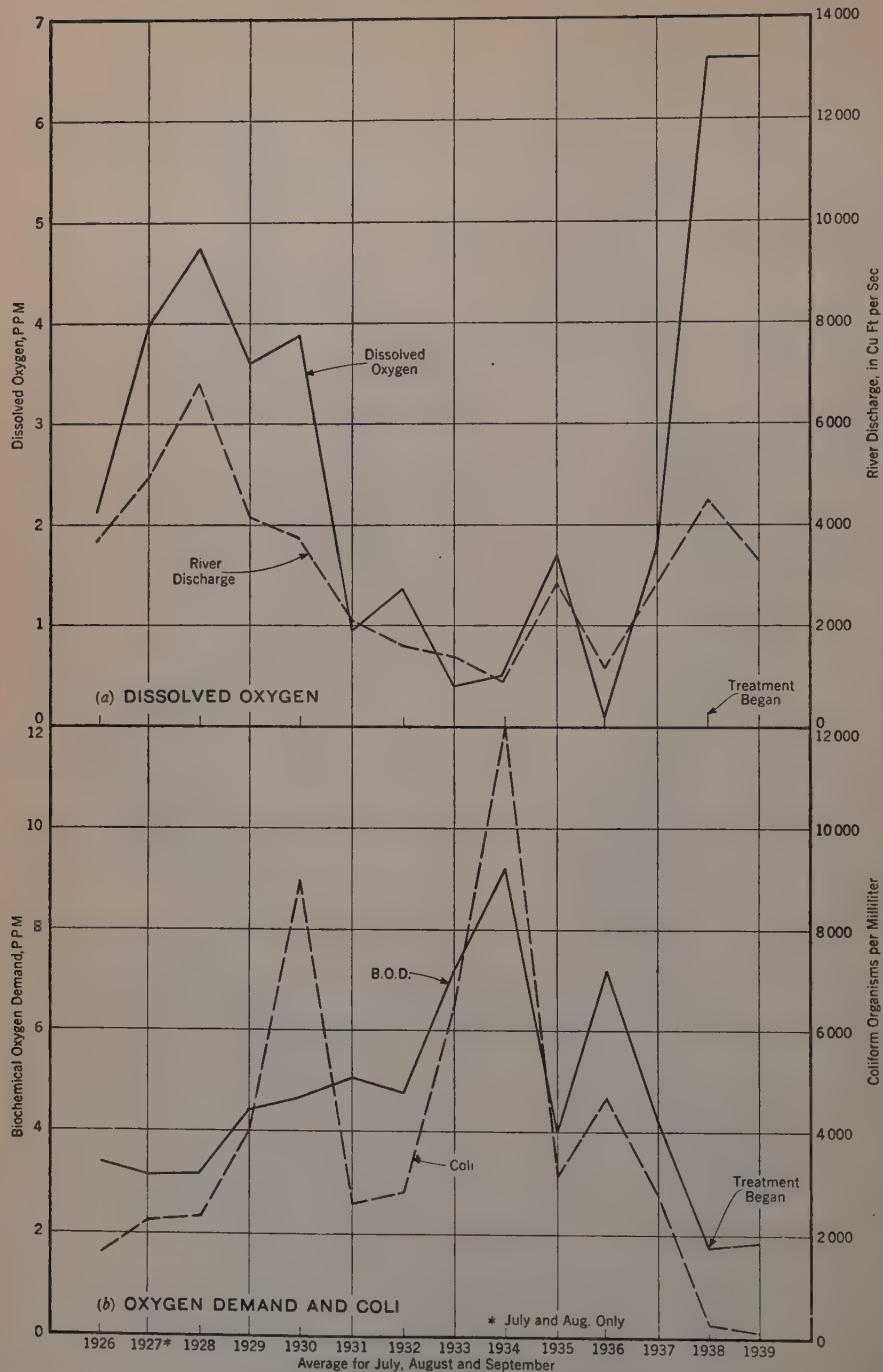


FIG. 3.—RIVER CONDITIONS AT STATION M 5.4, TWIN CITY LOCK AND DAM, BEFORE AND AFTER SEWAGE TREATMENT

old sewer outlets, occasional leakage from the latter, unaccounted-for sources of pollution, and some residual effect from the old sludge deposits in this area.

TABLE 11.—AVERAGE ANALYTICAL DATA FOR JULY, AUGUST, AND SEPTEMBER, 1937 AND 1939, MISSISSIPPI RIVER, THROUGH THE TWIN CITIES

Station (see Fig. 2)	1937				1939			
	PPM		COUNT PER ML		PPM		COUNT PER ML	
	Dissolved oxygen	B.O.D.	Total bacteria	Coliform organisms	Dissolved oxygen	B.O.D.	Total bacteria	Coliform organisms
M 0-4.8	7.85	1.55	1,700	2.0	7.40	1.45	3,000	4.0
M 0.3	6.10	4.25	340,000	1,100	7.65	1.65	5,000	42
M 7.5	0.80	4.20	340,000	2,000	6.65	1.85	11,000	110
M 13.8					6.65	2.10	12,800	150
M 16.6					6.60	2.20	9,200	110
M 17.3	1.25	3.50	210,000	2,100				

A more complete picture of the effect of the operation of the Twin Cities' plant on river conditions is shown in Table 12, which contains data on dissolved

TABLE 12.—A COMPARISON OF MISSISSIPPI RIVER FLOWS (CU FT PER SEC) AND DISSOLVED OXYGEN CONTENT (PPM)
(Average for July, August, and September 1931-1939, Inclusive)

Stations ^a	DISSOLVED OXYGEN (PPM)								
	1931	1932	1933	1934	1935	1936	1937	1938 ^b	1939
St. Paul ^c	2,430	2,140	1,715	1,090	3,382	1,476	4,000	7,740	4,000 ^d
M 0-4.8	6.65	7.55	7.50	8.35	7.55	9.05	7.85	8.40	7.40
M 0.3	5.05	4.90	4.60	1.75	6.15	2.70	6.10	7.50	7.65
M 5.4	0.95	1.35	0.40	0.50	1.70	0.10	1.75	6.60	6.60
M 7.5				0.40	1.65	0.15	0.80	6.85	6.65
M 13.8								6.85	6.65
M 16.6								6.00	6.60
M 17.3-18.0	0.70	1.40	0.25	0.35	1.30	0.80	1.25	5.55	5.05
M 22.7	0.60	0.80	0.30	0.20	1.05	0.75	1.25	3.35	2.00
M 26.5					1.80	2.65	1.35	2.90	1.25
M 30.5					2.95	3.35	2.55	2.60	1.95
M 33.9								3.10	3.25
M 37.8	1.45	3.90	2.95	5.25	4.85	5.95	5.30	4.60	6.00
M 39.1	5.65	7.40	7.25	7.10	7.30	7.15	7.20	6.15	6.50
M 42.8							6.70	6.40	6.00
M 47.0							6.30	6.45	6.00
M 50.9							6.65	6.35	6.50
M 56.1							6.60	6.30	6.60
M 58.1								6.05	6.65
M 61.2	5.10	6.60	6.15	6.75	6.05	5.90	6.60	6.05	6.85
M 74.0	6.35	8.50	7.00	7.35	7.20	7.90	7.00	6.70	6.80
M 80.2	6.55	7.50	8.00	8.10	9.45	6.60	7.10	6.30	6.25
M 88.1	6.80	7.15	6.30	7.55	8.15	7.05	6.35	6.85	5.80
M 92.6	6.65	7.20	6.35	7.35	7.30	6.70	5.80	6.40	6.15

^a Indicates mileage from the head of navigation at the lower Northern Pacific Railway bridge in Minneapolis—see Fig. 2. ^b Twin Cities' sewage treatment plant was placed in operation during the summer of 1938. ^c River flow at St. Paul. ^d Estimated from flow at Lock and Dam No. 1.

oxygen concentration at various sampling stations in a 100-mile stretch of the Mississippi River. The location of the sampling stations in relation to the Twin Cities and the treatment plant is shown in Fig. 2. Referring to Table 12, it will be seen that oxygen concentrations through the Twin Cities (stations

M 0-4.8 to M 16.6) are, as would be expected, appreciably higher than before the sewage was intercepted and treated. Immediately below the plant the dissolved oxygen dropped to 5.05 ppm, and at a point 10 to 14 miles below the Minneapolis-St. Paul plant (9 miles below South St. Paul and Newport) dropped to its low average value of 2.60 ppm in 1938 and 1.25 ppm in 1939. The standard tentatively fixed for this section of the river (pool of the Hastings Dam) is 2 ppm of dissolved oxygen during the summer months. Below this point the river quickly recovered, so that at Hastings, approximately 22 miles below the Twin Cities' treatment plant, the water contained more than 6 ppm of oxygen.

In the winter periods, critical conditions formerly prevailed during January, February, and March. This was from the standpoint of fish life below the confluence with the St. Croix River, for which a tentative standard of 4 ppm of dissolved oxygen has been set. Table 13 contains comparative data for

TABLE 13.—DISSOLVED OXYGEN CONTENT OF MISSISSIPPI RIVER IN FEBRUARY OF VARIOUS YEARS (PPM)

Station	1935 ^a	1939 ^b	1940 ^c	Station	1935 ^a	1939 ^b	1940 ^c
M 0-4.8 River entering Minneapolis.....	6.1	6.3	7.5	M 39.1 Hastings Bridge.....	7.2		
M 0.3.....	6.9	7.6	8.7	M 42.8 Below junction with St. Croix River.....	8.0	6.9	
M 5.4.....	4.5	7.9	8.8	M 47.0.....	7.8	6.3	
M 13.8.....	3.3	6.7	8.0	M 50.9.....	7.6	5.8	5.9
M 16.6 Twin City sewage treatment plant.....	2.7	6.9	7.3 ^d	M 56.1 Red Wing Dam.....	7.3	6.2	6.0
M 20.0 South St. Paul.....	1.8	6.6		M 61.2.....	7.0	7.8	6.5
M 22.7.....	1.2	6.3	5.3	M 74.0 Frontenac.....	4.5	6.9	6.2
M 30.5.....	0.7	4.5	1.0	M 80.2 Lake City.....	3.3	6.9	9.0
M 37.8 Hastings Dam.....	0.4	3.2	0.3	M 88.1 Outlet of Lake Pepin.....	3.5	7.0	14.0
				M 92.6 Wabasha.....		7.9	

^a Flow at St. Paul, February, 1935, 1,613 cu ft per sec. ^b Flow at St. Paul, February, 1939, 3,730 cu ft per sec. ^c Flow at St. Paul, February, 1940, 1,400 cu ft per sec. ^d Station M 18.0.

February, 1935, and February, 1939 and 1940. February, 1935, was selected because during that month the river flow of 1,613 cu ft per sec at St. Paul was extremely critical. Instead of 4 ppm below the St. Croix River, as tentatively set, individual samples collected in Lake Pepin have been practically devoid of oxygen at various times. In February, 1940, with flows of 1,400 cu ft per sec, the lowest individual sample of dissolved oxygen below the St. Croix River was 4.5 ppm, and the average minimum for the month was 5.9 ppm. During this period the Minnesota River entered the Mississippi River with an average of only 1.5 ppm (lowest 0.6 ppm) of oxygen, and the packing plants at South St. Paul and Newport were enjoying an unusually heavy volume of business with a consequent heavy pollution load. The river flow at St. Paul in February, 1940, was the lowest winter flow, with the exception of one month, than at any time in the 53-yr period of record.

River Bottom Conditions.—In October, 1939, a study was made of the river bottom conditions in that part of the Hastings pool below the Twin Cities' sewage treatment plant, a distance of 21 miles. Cross-section soundings were made at 22 stations and composite bottom samples collected at 14 stations. It

was found that this stretch of river divided itself into three sections, on which some of the pertinent data are summarized in Table 14.

Section (A) is a 2-mile stretch of river extending from a point immediately upstream from the Twin Cities' sewage plant to a point just upstream from the

TABLE 14.—RIVER BOTTOM STUDY (TREATMENT PLANT TO HASTINGS DAM)

Section	CROSS-SECTIONAL WATER AREA, IN SQ FT		RELATIVE RIVER VELOCITY		5-DAY B.O.D. OF BOTTOM MATERIAL, IN PPM		Physical character of bottom material
	Aver- age	Maximum	Aver- age	Mini- mum	Aver- age	Maximum	
(A) M 16.6-18.5	5,800	7,900(M 16.8)	1.0	1.0	630	690(M 16.8+380 ft)	Dirty sand and gravel Considerable sludge and silt Some sludge and silt deposits
(B) M 18.5-26.5	9,600	15,100(M 20.5)	0.6	0.5	1,050	1,650(M 19.5)	
(C) M 26.5-37.5	15,000	28,000(M 35.5)	0.4	0.3	820	895(M 28.5)	

South St. Paul packing plants. The data showed that the river bottom through this section was of uniform character, being mostly sand and gravel with just enough organic matter to give the comparatively low average B.O.D. of 630 ppm. The bottom was quite clean, with practically no sludge deposits. Such material as existed may originate from storm runoff and overflow from sewers during storms, by-passing of excess storm flows at the sewage treatment plant, general river silt (of which the Minnesota River contributes a large proportion), and any suspended solids remaining in the treatment plant effluent.

Section (B) is an 8-mile stretch of river extending from a point just upstream from the South St. Paul packing plants to a point 4 miles downstream from the Invergrove Bridge. This is the section most affected by the packing plant wastes in respect to bottom deposits. There are considerable deposits in this section which exert a large oxygen demand on the overlying water. The oxygen demand of the bottom samples from this section was about twice as great as in section (A) and does not appear to have decreased much, if any, in comparison with recent years. The untreated sewage and packing wastes of South St. Paul and Newport discharging into the upper part of this section no doubt have contributed heavily to these deposits. In general, the river bottom in section (B) may be described as sand and gravel covered to a considerable extent with deposits of sludge, silt, and straw.

Section (C) is an 11-mile stretch of river extending from a point 4 miles downstream from the Invergrove Bridge to the Hastings Dam. This section contains many large but shallow backwater areas more or less cut off from the main river and not believed to affect the channel velocity appreciably. In general, the river bottom in this section may be described as mostly sand, gravel, and rock, covered to some extent with deposits of sludge and silt, which exert a somewhat smaller oxygen demand on the overlying waters than in section (B) but greater than in section (A). The oxygen demand of the bottom material in this section appears to be about the same as in 1937, the last previous year of complete cross-sectional sampling in this area. The oxygen demand exerted

by the sludge deposits in sections (B) and (C) tends to reduce the dissolved oxygen in the overlying waters very appreciably and correspondingly affects the maintenance of the desired minimum dissolved oxygen in this part of the river.

General.—It is unfortunate that it is impossible to make a complete evaluation of the recovery of the Mississippi River through, and downstream from, the Twin Cities from its former heavy pollution load because of the fact that comparable treatment is not being provided by downstream communities. The writer has looked forward anxiously to the time when the original river calculations, on which the degree of treatment decided upon was based, could be checked by actual observations and a determination made of several factors affecting deoxygenation and re-aeration after the system was in operation. Such preliminary calculations as have been made, substantiated by analytical data and observation, indicate that the original assumptions, if anything, were conservative, and that the type of treatment selected will meet maximum requirements and at the same time will permit variations of treatment provided with river dilution conditions and requirements.

ACKNOWLEDGMENT

The Minneapolis-St. Paul Sanitary District is under the direction of a Board of Trustees of seven members. In charge of plant operation are Clifford R. Raiter, John C. Sager, Warren H. Sleeper, Assoc. M. Am. Soc. C. E., and Benjamin M. Storey, plant operators. Kerwin L. Mick is chief chemist in charge of the laboratory, and Stanley J. Larsen is mechanical engineer in charge of maintenance. The writer wishes to acknowledge the fine cooperation of all the members of his staff, as well as of superintendents of other plants and of other engineers engaged in this field, the effect of whose assistance and free interchange of information and experience has been a constant source of encouragement in the operation of this plant as it has in many others.

CONCLUSION

The foregoing review of experiences in the first two years of operation of the Twin Cities' plant should be considered as in the nature of a progress report. So many interesting possibilities of producing economies and effecting simpler and even better operation have presented themselves that operations here cannot be considered to be on a standardized basis. As a matter of fact, it is the writer's hope and expectation that operation will not be considered to have reached that point for some time, but rather that the possibilities of new developments will be continually explored. As many plant operators will agree, it is these possibilities that make operation of a sewage treatment plant, over a period of years, a succession of interesting problems, rather than a repetition of routine and standardized tasks.

RIGID FRAMES WITHOUT DIAGONALS (THE VIERENDEEL TRUSS)

BY LOUIS BAES,¹ ESQ.

SYNOPSIS

Attention is called herein to a simple method for the design of Vierendeel trusses with chords of equal moment of inertia, the loads being applied only at the joints. The method is based on the fact that, for these trusses, a point of contraflexure exists in each vertical and that, according to theory checked by photoelasticity, this point is at the midheight of the vertical.

INTRODUCTION

Until recently, the Vierendeel truss has been adapted mostly to designs of highway bridges, railway bridges, steel framed towers, and the structural steel frames of buildings. There is now a tendency to extend this range to the design of cranes, locomotives, steel passenger cars, etc. This type of truss also has possibilities in the design of reinforced concrete. Among the examples of its use in the United States may be cited a group of eight steel bridges, all of the Vierendeel type, constructed in 1936 in Los Angeles, Calif. Since designers in the United States are showing more and more interest in this type, the time seems favorable to open the subject for technical discussion.²

HISTORICAL

The Vierendeel Truss Before 1914.—The concept of the simple rigid truss without diagonals (variously termed the "ladder" truss, or the "arcade" truss) is due to Prof. Arthur Vierendeel. In 1893, he adopted this type for the steel steeple of the church of Dadizeele, in Flanders, Belgium. In Tervueren, near Brussels, Belgium, on the occasion of the International Exhibition of Brussels in 1897, Professor Vierendeel designed a riveted steel test bridge, 103 ft long, having two main girders of uniform depth divided into nine panels without diagonals. The verticals were joined to the chords by large curved gussets. The angles that were used as the outer flanges of the vertical members were

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by May 15, 1941.

¹ Prof. of Stability, Univ. of Brussels, Brussels, Belgium.

² *Transactions, Am. Soc. C. E.*, Vol. 102 (1937), p. 869.

bent, as shown in Fig. 1, to become the inner flanges of the chord members. When it was subjected to load tests,³ rupture occurred in the panels near the very end of the bridge. Even during this early period, Professor Vierendeel was actively promoting the general acceptance of this type of truss.^{4,5}

The introduction of this type of truss without diagonals was such a distinct departure from the accepted practice of the time that it aroused considerable

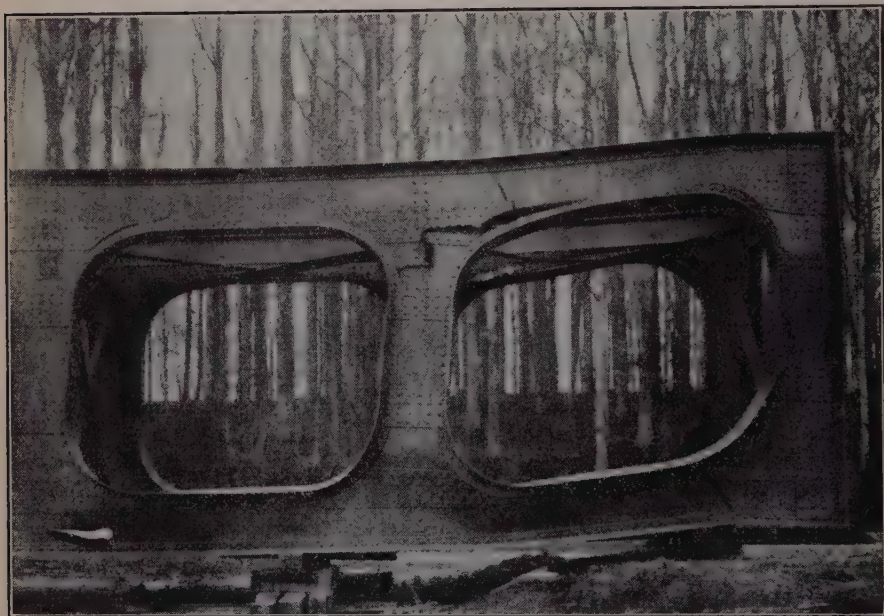


FIG. 1.—END PANELS OF EXPERIMENTAL VIERENDEEL BRIDGE TESTED TO FAILURE AT TERVUEREN (BELGIUM) IN 1897

controversy in Belgium. Practically all engineers were radically opposed to the system, some on general principles, and others for reasons of economy or efficacy.

It is true that, architecturally, these early trusses, for the most part with parallel top and bottom chords, were not attractive. Furthermore, it seems that it might be difficult to rivet a large curved gusset plate at the joint.

Except for the experimental bridge at Tervueren, the first steel highway bridge of this type was built in 1902 at Avelghem, on the Schelde River, with a span of 138 ft. The first steel single-track railway bridge of this type was built, over the Lys River, at Grammene, Belgium, in 1923. The span length was 185 ft and the structure was subjected to systematic load tests recorded

³"Le Pont système Vierendeel: Expériences de Tervueren," by A. Vierendeel, Bruges, 1898; see, also, *Annales des Travaux Publics de Belgique*, February, 1898.

⁴"Longerons en treillis et longerons à arcades," by A. Vierendeel, Brussels, 1897; and "Théorie Générale des Poutres Vierendeel," by A. Vierendeel, *Mémoires de la Société des Ingénieurs Civils de France*, August, 1900.

⁵"Les ponts architecturaux en métal," by A. Vierendeel, *Annales des Travaux Publics de Belgique*, October, 1896.

in 1925.⁶ The first reinforced concrete footbridge was constructed in La Louvière in 1911 with a span of 184 ft. The upper chord was arched and intersected the lower chord at the end of the span.

Prior to 1914, six steel highway bridges were constructed in Belgium. One drawbridge had been designed for Bruges, but was not constructed until 1920. Each of these steel structures had riveted connections. In the first type (the Tervueren Bridge) many of the members had to be cambered; that is, as previously stated, the flanges of the verticals were bent so as to serve as the inner flanges of the chords. No other bridge of this exact type was ever built in Belgium. In subsequent designs, Professor Vierendeel introduced an important simplification permitting a more rational solution; that is, he made the chords continuous throughout. In general, the verticals were made of constant cross section, the flanges being extended up against the chord flanges, the joints being stiffened by angles or by large gusset plates. In other cases, the verticals were made gradually deeper toward the chords, thus forming the large connecting gussets at the joints.

Development Since 1920.—From 1920 to 1939, and more especially from 1928, there has been a marked increase in the number of Vierendeel trusses built in Belgium and in the Belgian Congo. For example, there are about twenty such structures in the Congo, among which may be cited Bukama Bridge with four 202-ft spans. Belgium has more than seventy Vierendeel bridges.

These structures differed, however, from the unattractive types created before 1914, and, following the concept of the bowstring truss with tie rods, most of the Vierendeel bridges built after 1920 were designed with curved top chords (second-degree parabolas) with the height of the end verticals theoretically reduced to zero.

Sixty such highway bridges were built in Belgium between 1928 and 1937. In the list are included two drawbridges and six steel railway bridges, with a few examples of reinforced concrete structures. Examples are shown in Figs. 2, 3, and 4.

CHARACTERISTICS OF THE VIERENDEEL TRUSS DESIGNED FOR BRIDGES

From experience with Vierendeel bridges in Belgium and in the Belgian Congo, the writer has noted the following characteristics favorable to the adoption of the Vierendeel truss with the curved upper chord:

(1) When they are suitably planned and skilfully designed (especially the gusset plates connecting the verticals to the chord members, and the shape of the ends), this truss may be made very attractive architecturally (Figs. 2 and 3).

(2) Under dead load this truss does not react as a Vierendeel truss, but as an ordinary bowstring arch. Consequently, under dead load, the various members are bent very little, if any, and are only slightly stressed.

(3) It is only under moving live loads that the structure really acts as a Vierendeel truss. The maximum bending moment in this truss will be found to be less than in the tied arch.

⁶"Essais de réception du pont-rail de Grammene," by A. Vierendeel, *Annales des Travaux Publics de Belgique*, April, 1925.



FIG. 2.—THE EYGENBILSEN BRIDGE, BUILT IN 1935; TOTAL LENGTH, 369 FT;
AND LENGTH OF MAIN SPAN, 211 FT



FIG. 3.—THE DUDZEELE BRIDGE, BUILT IN 1935; SPAN, 104 FT



FIG. 4.—DRAWBRIDGE OF MUIDE (GHENT, BELGIUM), BUILT IN 1933; CHANNEL SPAN,
68 FT; AND COUNTERWEIGHT SPAN, 43 FT 6 IN.

(4) Analyzed from the viewpoint of items (1) to (3), the curved Vierendeel truss will often be found more economical than the ordinary tied arch.

(5) Compared with the conventional steel tied arch, the Vierendeel truss inherently has greater stiffness which is a valuable characteristic especially in railroad bridges constructed for heavy and fast traffic.

(6) It is logical and desirable to make both chords continuous. The verticals are simply attached to the chords by means of large connecting plates.

(7) It is logical in principle to design the upper chord with the same section as the lower chord, provided the floor beams are attached only at panel points. Furthermore, it is convenient, in general, to design all the verticals with the same cross-sectional area. These conditions will help to simplify problems of design as well as construction.

(8) As compared with the Vierendeel truss having a curved upper chord, the parallel-chord type is seldom attractive in appearance. The latter should be adopted only in cases where the uniform height is required by the conditions of the design. Furthermore, the stress distribution in the parallel-chord type is much less favorable than in the arch type, particularly in the end panels where the shearing forces are extremely high. Where the parallel chord is necessary, this condition at the end panels can be met by spacing the vertical members closer together near the ends of the truss, as suggested by J. D. Gedo,⁷ M. Am. Soc. C. E. The designer, however, might encounter difficulties at least in applying this principle to bridge structures.

(9) With the assistance of arc welding performed with care and skill, the Vierendeel truss becomes a sturdy structure with simple and sober lines, permitting easy maintenance and offering little encouragement to corrosion.

PRINCIPLES OF DESIGN FOR TRUSSES WITHOUT DIAGONALS

Undoubtedly, the greatest obstacle to the general acceptance of the truss without diagonals has been (and still is) its reputation for including certain design assumptions and for being difficult to design.

In the present state, however, the design of trusses without diagonals, having two chords of equal reduced moment of inertia, $I \cos \alpha$, and receiving the loads only at the joints, is as easy as the design of any other type of truss.

Point of Contraflexure in the Verticals.—From the very beginning of his studies, Professor Vierendeel has stated that the design was predicated upon the fact that there must be a point of contraflexure in each vertical of the truss. When the upper and the lower chords are parallel and have the same moment of inertia, Professor Vierendeel assumed this point to be at the midheight, regardless of the loading, with the assumption that the loads were applied only at the joints. For two chords that are not identical he assumed that the bending moment taken by them was proportioned in the ratio of their moments of inertia, at the sections under consideration. For the tied arch type of truss, he sometimes assumed (to simplify the design) that the points of contraflexure were at the top of the verticals.

⁷ Transactions, Am. Soc. C. E., Vol. 102 (1937), p. 933.

Professor Vierendeel and later writers⁸ have attempted to demonstrate that when the parallel upper and lower chords are not of equal cross sections, the point of contraflexure in the verticals occurs in the ratio, $\frac{h_u}{h_l} = \frac{I_u}{I_l}$, in which I_u and I_l are the moments of inertia of the upper and the lower chords, respectively, and the height, h , between the chord axes, is divided, by the point of contraflexure, into an upper section, h_u , and a lower section, h_l . This assumption has been disproved, but it continues to mislead many designers. It is important to emphasize this fact because advocates of this theory claim to have checked it and, thereafter, apply it erroneously but confidently. Other writers have claimed that $\frac{h_u}{h_l} = \sqrt{\frac{I_u}{I_l}}$, a relation which is likewise incorrect.⁹

Still other writers, including Professor Vierendeel himself, have tried to locate the points of contraflexure in the chord members; and they advance

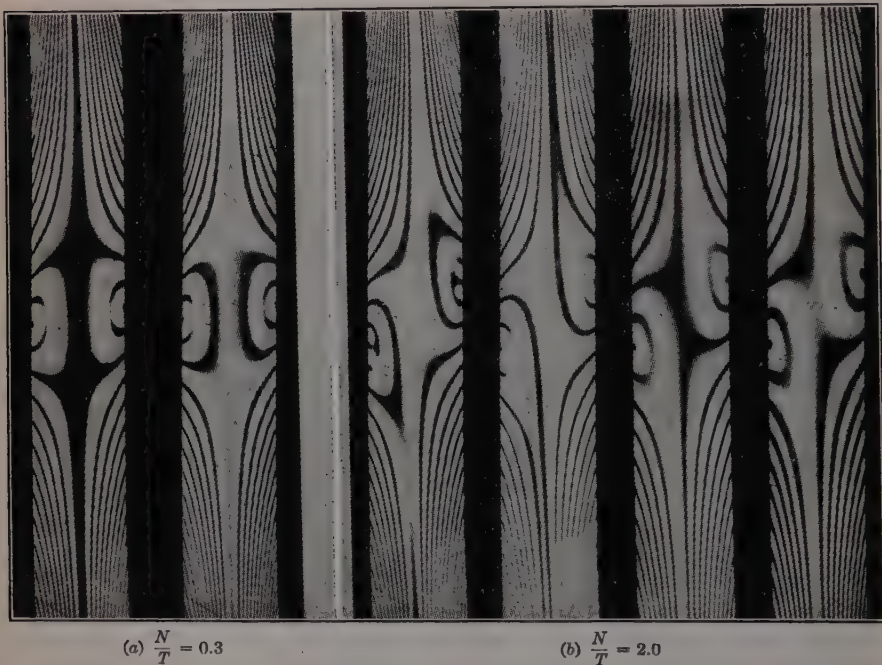


FIG. 5.—PHOTOELASTIC VIEWS SHOWING STRESS PATTERNS IN THE REGION OF THE POINT OF CONTRAFLEXURE OF A STRAIGHT SPECIMEN SUBJECT TO PLANE BENDING AND AXIAL LOAD

systems of design based on this principle.¹⁰ It seems clear now, however, that the points of contraflexure that may occur in the chord members are not fixed points. Their location is variable and depends on loading conditions. Moreover, in the arch type of upper chords, there may be no points of contraflexure

⁸ *Transactions, Am. Soc. C. E.*, Vol. 102 (1937), pp. 908-910.

⁹ "Der Eisenbetonbau," by R. Saliger, 6 Aufl., Leipzig, 1933.

¹⁰ *Transactions, Am. Soc. C. E.*, Vol. 102 (1937), pp. 901-907.

in the chord. Consequently, it is impossible to establish generalized reliable methods of design based on locating points of contraflexure in the chord members. It would be necessary to establish a separate design system for each condition of loading or to obtain a rough approximation.

The controversy has been greatly aided by the introduction of photoelasticity as a laboratory method of analyzing structures. By this means, checked by analytical computations (which agree remarkably well), it has been found that in monochromatic light the lines of extinction in the vicinity of the points of contraflexure are quite characteristic.

Fig. 5 shows two photoelastic stress patterns in the region of a point of contraflexure in a straight specimen subjected to plane bending combined with

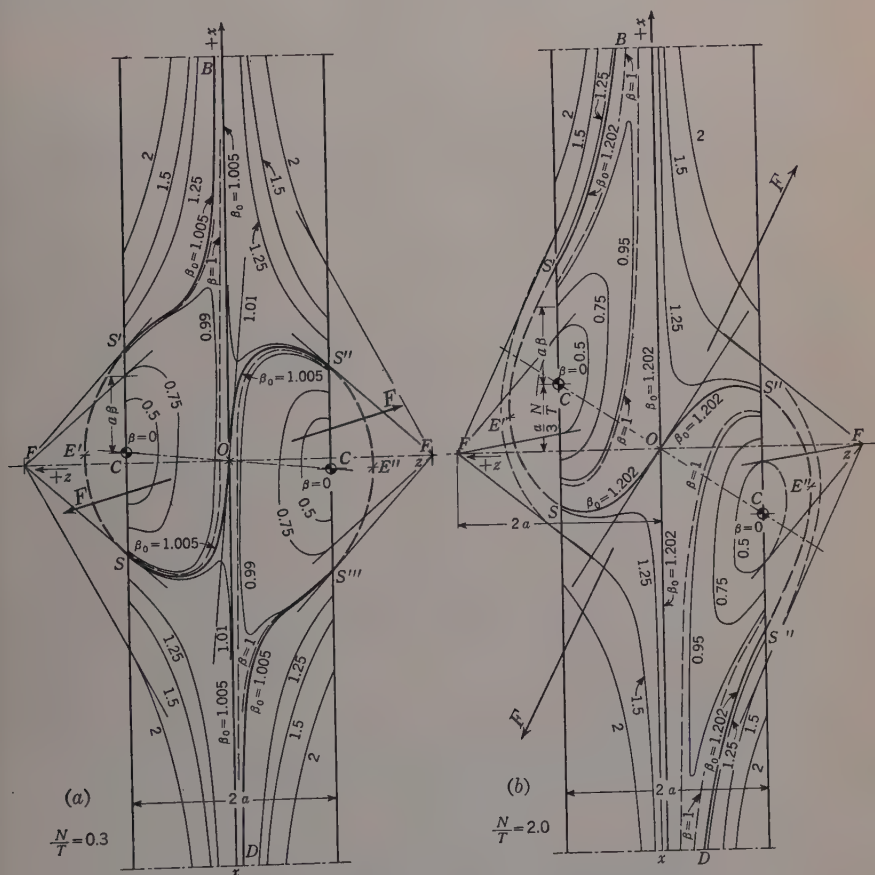


FIG. 6.—GRAPHICAL REPRESENTATION OF THE STRESS PATTERNS IN FIG. 5 AS PLOTTED FROM CORRESPONDING THEORETICAL FORMULAS

axial stress. The ratios are $\frac{N}{T} = 0.3$ and 2.0 , respectively, in which N = the axial force and T = the shearing force. This photoelastic test was made with monochromatic light. The black curves are extinction lines, or curves of equal

tangential maximum stress. Their number depends upon the intensity of the loads applied.

The equations of these curves have been completely derived, and are represented¹¹ graphically in Fig. 6 for the two cases shown in Fig. 5. It will be noticed how closely these curves check with the experimental photographs.

An extensive program of experiments, covering a range of cases beyond all possible practical requirements, has enabled the writer to confirm the following expression for fixing very approximately the position of the points of contraflexure in vertical members when they all have the same cross section and all panels have the same length:

$$\frac{h_u}{h_l} = \frac{\frac{l}{h} \times \frac{I_v}{I_l} + C}{\frac{l}{h} \times \frac{I_v}{I_u} + C} \dots \dots \dots (1)$$

in which l = length of panel; I_v = moment of inertia of the vertical; and C is a constant equal to 2.5 in the end verticals and 6.0 in all intermediate verticals. For the case of non-parallel chords I_u and I_l must be replaced by

$$I_u' = I_u \cos \alpha_u \dots \dots \dots (2a)$$

and

$$I_l' = I_l \cos \alpha_l \dots \dots \dots (2b)$$

in which I_u and I_l = the moment of inertia of the non-parallel upper and lower chords, respectively; and α_u and α_l = the corresponding angles between the chords and the horizontal. These expressions— $I \cos \alpha$ —are called the reduced moments of inertia of the members. In both diagrams F is the resultant external force, the components being N along axis x and T along axis z .

Eq. 1 was first presented by P. Thomas in 1924,¹² and later verified as being practically accurate by the writer.¹¹ In most cases encountered in practice, the ratio, $\frac{h_u}{h_l}$, is so nearly constant for both end verticals and intermediate verticals that $C = 6.0$ may be used for all cases. In other words, today, it has been adequately established that for a given value of the ratio, $\frac{I_l}{I_u}$ or $\frac{I_l'}{I_u'}$, the positions of the points of contraflexure in vertical members of a Vierendeel truss depend on the quantities, $\frac{l}{h} \times \frac{I_v}{I_u}$ or $\frac{l}{h} \times \frac{I_v'}{I_u'}$. When the chord members have the same reduced moment of inertia $I_u' = I_l'$, Eq. 1 indicates that the points of contraflexure are always at the midheight of the verticals. The points move upward if: (a) The ratio $\frac{I_l}{I_u}$ or $\frac{I_l'}{I_u'}$ increases; (b) the verticals become stiffer (that is, if $\frac{I_v}{I_u}$ increases); and (c) the ratio $\frac{l}{h}$ increases.

¹¹ "La poutre Vierendeel," by L. Baes, *L'Ossature Métallique*, October, 1936, Brussels; also, "Étude de la région voisine des points d'inflexion d'une pièce droite prismatique sollicitée par flexion plane composée, avec effort tranchant," by L. Blanjean et F. Temmerman, *Prize Paper, Bulletin de la Société Royale Belge des Ingénieurs et des Industriels*, Brussels, 1937.

¹² "Le calcul des poutres Vierendeel," by P. Thomas, *Revue Universelle des Mines*, Liège, Belgium, November 15, 1924; see, also, issues of August 1, 1926, and November 15, 1926.

Although, theoretically, the position of the point of contraflexure depends on the condition of loading for all ordinary cases, it is practically independent of it if the loads are applied only at the panel points. Moreover, it is only for higher values of the quantity, $\frac{l}{h} \times \frac{I_v'}{I_u'}$, that the points of contraflexure are at an appreciable distance from the center. For all ordinary cases, furthermore, the points of contraflexure in verticals of a Vierendeel truss, carrying no loads except those at the panel points, are at midheight, or very nearly so. The foregoing questions, which have been subjects of discussion since before 1900, have now been solved by analytical methods checked by photoelasticity.

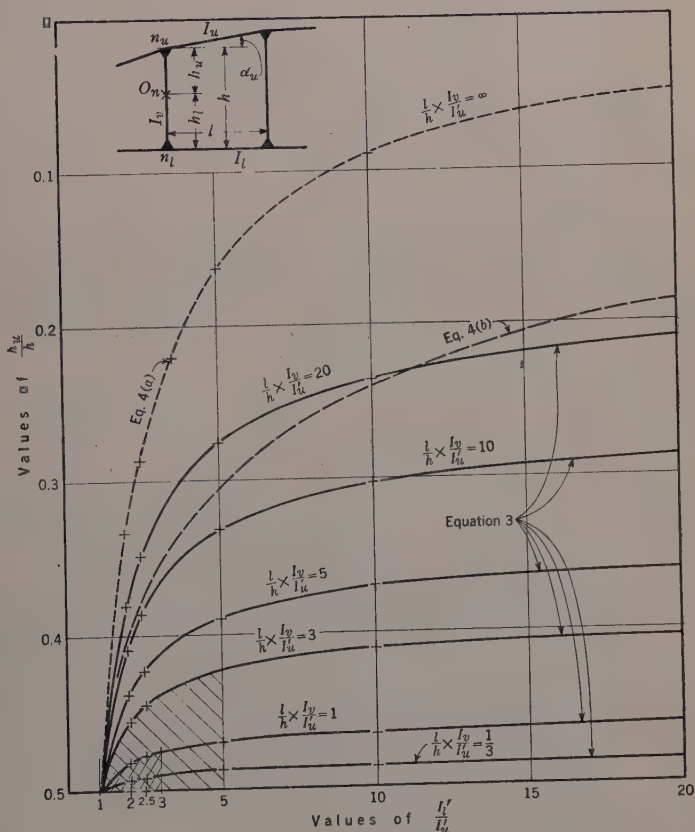


FIG. 7.—COMPARISON OF FORMULAS GIVING POSITION OF POINT OF CONTRAFLEXURE IN THE VERTICALS

This determination of the location of the points of contraflexure in the verticals, or of the distribution of the bending moment between the two chords, is essential. In fact, it makes it possible to determine by experiment whether a design method is accurate or not. Fig. 7 shows the considerable difference

between the results given by Eq. 1:

$$\frac{h_u}{h_l} = \frac{\frac{l}{h} \times \frac{I_v}{I_n'} + 6}{\frac{l}{h} \times \frac{I_v}{I_l'} + 6} \dots \dots \dots (3a)$$

or

$$\frac{h_u}{h_l} = \frac{\frac{l}{h} \times \frac{I_v}{I_n'} + 6}{\frac{l}{h} \left(\frac{I}{I_u'} + \frac{I}{I_l'} \right) + 12} \dots \dots \dots (3b)$$

and by the formula derived by Professor Vierendeel and others:

$$\frac{h_u}{h_l} = \frac{I_u'}{I_l'} = \frac{I_u \cos \alpha_u}{I_l \cos \alpha_l} \dots \dots \dots (4a)$$

or Prof. R. Saliger's formula

$$\frac{h_u}{h_l} = \sqrt{\frac{I_u'}{I_l'}} \dots \dots \dots (4b)$$

Most cases of Vierendeel trusses fall within the cross-hatched area of Fig. 7 and almost always in the very small area that is double-crossed. It will be noticed how inaccurate Eqs. 3 are; yet they are (especially Eq. 3a) the basis of many design methods for Vierendeel trusses with chords having different reduced moments of inertia (see Figs. 5 and 6).

THE MODERN METHOD OF ANALYSIS FOR TRUSSES WITH CHORDS HAVING THE SAME REDUCED MOMENT OF INERTIA AND WITH LOADS APPLIED ONLY AT THE JOINTS

It has been difficult to analyze a Vierendeel truss because, if N is the number of panels, the computations involve $3N$ internal hyperstatic unknowns. Frequently, there are from eight to twelve panels and the number of internal hyperstatic unknowns in such cases will vary between twenty four and thirty six. It is clear that if the computations are not organized properly and skilfully the analysis will be long, tedious, and subject to error. In 1897, Professor Vierendeel,¹³ sensing the inherent importance of the points of contraflexure in the posts of his truss, presented a design method based upon the principle of dividing the truss horizontally by cutting each vertical at its point of contraflexure (see Fig. 8). The number of internal hyperstatic unknowns by this method is reduced at once to $2(N+1)$, connected by two statical conditions of equilibrium.

In the thirty years since about 1908, however (especially since the introduction of reinforced concrete), many hyperstatic systems with rigid connections have been introduced and designers have developed many ingenious methods of analysis. The Vierendeel truss, of course, has not been ignored in this development. Unfortunately, methods applying to the analysis of the Vierendeel truss in general have been such as to confirm a popular opinion that the

¹³ "Longerons en treillis et longerons à arcades," by A. Vierendeel, Brussels, 1897.

computations are long, difficult, and, consequently, subject to inadvertent errors. In the United States, for example, certain methods of analyzing this truss have been proposed which are very remarkable from an analytical point of view but which do not simplify the problem.

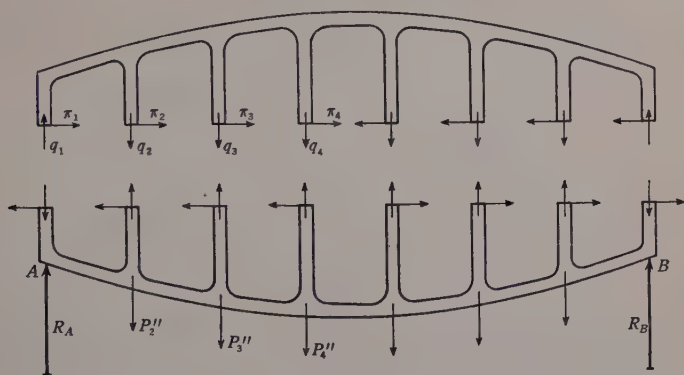


FIG. 8.—ISOSTATIC SYSTEM OF REFERENCE USED BY PROFESSOR VIERENDEEL

The writer has shown¹⁴ that the simplest method of analyzing a Vierendeel truss involves choosing as a principal or isostatic system of reference the arrangement shown in Fig. 9. The upper chord is cut at the center of each panel.

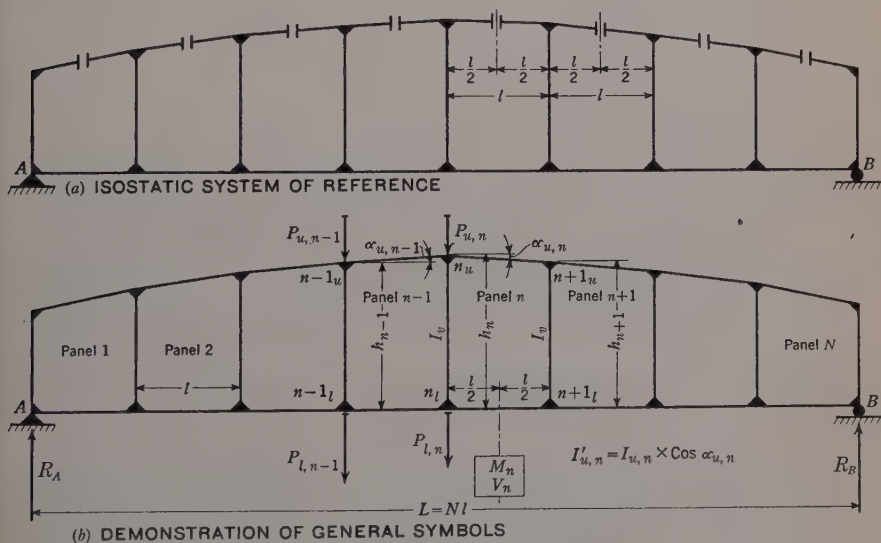


FIG. 9.—DESIGN SKETCHES

Since the two chords have the same reduced moment of inertia, and since the loads are only applied at the panel points, the points of contraflexure in the verticals are at midheight.

¹⁴ "La poutre Vierendeel," by L. Baes, *L'Ossature Métallique*, Brussels, October, 1936, and March and September, 1937.

A method may be applied which was developed by K. Kriso¹⁵ and redemonstrated, in another manner, by the writer.¹⁴ An interpretation of general symbols is afforded by Figs. 9(b) and 10, in which M_n is the external bending moment; V_n is the external shearing force, at the center of panel n ; I_{un} , I_{ln} , and I_v are the moments of inertia of the cross sections, respectively, of the upper chord, the lower chord, and the verticals; O_n , O_{n+1} , etc., are the points

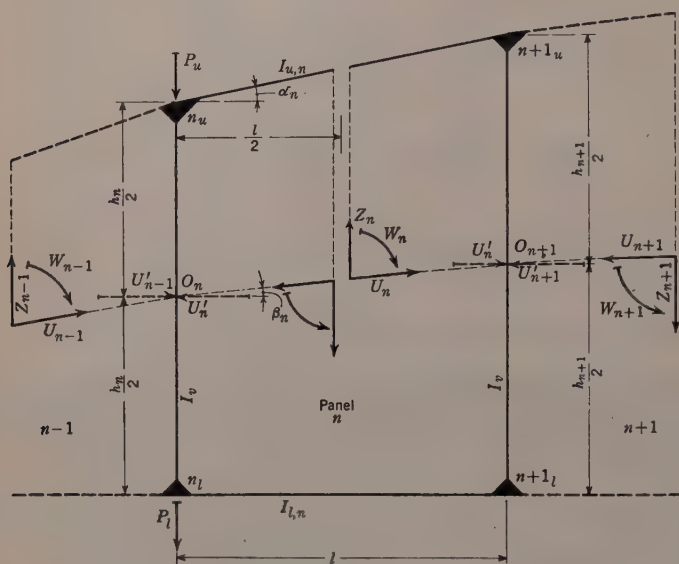


FIG. 10.—EXPLANATION OF SYMBOLS IN PANEL n

of contraflexure in the verticals; and W_n , Z_n , and U_n are the three hyperstatic unknowns in panel n . The designer selects, as hyperstatical unknowns in each panel, a torque, W_n , and two forces, Z_n and U_n (or the horizontal component, U'_n), and applies them all at the point shown in Fig. 10. For this arrangement, a group of three equations is derived for each panel which can be solved easily due to the essential property that the three hyperstatic unknowns in one of the panels affect only the four members constituting this panel and no other member.

For each panel, the following equations are obtained, for trusses with parallel chords:

$$-U_{n-1}' + \left(2 + 6 \frac{l}{h} \times \frac{I_v}{I_{un}}\right) U_n' - U_{n+1}' = 6 \frac{l}{h} \times \frac{I_v}{I_{un}} \times \frac{M_n}{h} \dots (5a)$$

$$W_n = \frac{1}{2} M_n \dots (5b)$$

and

$$Z_n = \frac{1}{2} V_n \dots (5c)$$

in which M_n and V_n are the external bending moment and shearing force acting on the truss at the midlength of panel n .

¹⁵ "Stabilité des poutres Vierendeel," by K. Kriso; translated from the German by E. Barbieux and E. Dryon, Paris, France, Béranger, 1926.

For trusses with non-parallel chords, W_n and Z_n are expressed by Eqs. 5b and 5c, but Eq. 5a is replaced by equation:

$$\begin{aligned}
 & -U_{n-1}' + \left\{ 1 + \left(\frac{h_{n+1}}{h_n} \right)^3 + \frac{3}{2} \times \frac{l}{h_n} \times \frac{I_v}{I_{u'n'}} \left(\frac{h_{n+1}}{h_n} + 1 \right)^2 \right. \\
 & \times \left[1 + \frac{1}{3} \left(\frac{h_{n+1} - h_n}{h_{n+1} + h_n} \right)^2 \right] \left. \right\} U_n' - \left(\frac{h_{n+1}}{h_n} \right)^3 U_{r+1}' = \frac{3}{2} \times \frac{l}{h_n} \\
 & \times \frac{I_v}{I_{l'n'}} \left(\frac{h_{n+1}}{h_n} + 1 \right)^2 \left(\frac{M_n + \frac{1}{6} \times \frac{h_{n+1} - h_n}{h_{n+1} + h_n} \times l \times V_n}{\frac{h_n + h_{n+1}}{2}} \right) \dots \dots (6)
 \end{aligned}$$

In the case under consideration, two of the hyperstatic unknowns in each panel (W_n and Z_n) become known at once, and it is virtually impossible to make an error in sign.

The entire problem is thus greatly simplified, being reduced to the simple task of solving the system of formulas of the same type as Eq. 5a or Eq. 6, which may be called the equations for the two or three U' -values of the panels separated by two adjacent verticals. For all cases, therefore, the solution involves a group of successive linear equations, in stages, starting with two variables, and involving as many formulas as there are panels. Such equations may be solved readily by a slide-rule, and they involve no complications due to successive approximations.

Eqs. 5 and 6 characterize the method and are as important in their connection as the three-moment equation (Clapeyron equation) for the moments in a continuous beam. Their solution is direct and simple as the writer has demonstrated in describing the design of a railroad bridge constructed in Belgium in 1937.¹⁶ In designing a tower only one assumption as to loading is required,¹⁷ and, in the case of a bridge, the method makes it easy to draw the influence line for every element of the computation.¹⁵ Kriso¹⁵ had considered it necessary to develop the equation for every single influence line. Although it is analytically quite remarkable, his study is difficult to read, and for that reason has not attracted attention. The direct solution of Eq. 5 or Eq. 6 for a limited number of positions of a single load is preferable; it demonstrates the simplicity and reliability of that method.

THE MODERN METHOD OF ANALYSIS FOR TRUSSES WITH CHORDS HAVING DISSIMILAR REDUCED MOMENTS OF INERTIA

The preceding method applies in the case when the chords have the same reduced moment of inertia and when the loads act only at the panel points. In all other cases, the method must be completed using the remarkable separating properties of the elastic centers of the panels. This new step was the subject of a thesis presented in 1939 by Emile Robert to the Faculty of Applied Sciences, University of Brussels. The problem is beyond the scope of the present paper; but the method can be used to solve all cases, without making

¹⁶ "La poutre Vierendeel," by L. Baes (second paper), *L'Ossature Métallique*, Brussels, March, 1937.

¹⁷ "La poutre Vierendeel," by L. Baes (third paper), *loc. cit.*, September, 1937.

any special assumption, and calling only on very simple first-degree equations, separating the three hyperstatic unknowns of each panel.

CONCLUSION

Personal experience with this subject leads the writer to conclude that:

(1) Regardless of the type of Vierendeel truss, and regardless of the use to which it is put, its analysis is simplest when a section is cut through one of the chord members between two verticals. This method is simple and altogether reliable.

(2) The method described in this paper is based essentially upon the assumption that the points of contraflexure in the vertical members may be determined precisely. Analytical investigation as well as experimental investigation reveal important characteristics pertaining to the location of these points. With the photoelastic method available, an exact theoretical solution can be checked.

(3) The method involves an equation applied to two or three panels which are separated by two adjacent verticals. A given case can be solved readily without successive approximations or other "trick" methods of computation. Such solution involves a complete analysis of the effects of live loads by means of influence lines.

(4) A preliminary design, which in itself is quite accurate, involves the analysis of only one or two conditions of loading.

(5) Thereafter, the computation of a Vierendeel truss, with chords having equal reduced moments of inertia and in which the loads are applied only at the panel points, is shown to be quite simple and free from too broad assumptions. This encourages the writer to believe that this type of truss, without diagonal members, will receive more attention on the part of structural designers in the future.

The writer desires, in this paper, merely to emphasize the existence of an accurate method of designing Vierendeel structures which is simpler, for example, than those currently in use in the United States.

ACKNOWLEDGMENT

The paper was translated into English by L. G. Rucquoi, M. Am. Soc. C. E., to whom the writer wishes to make sincere acknowledgment.

HYDRAULICS OF SPRINKLING SYSTEMS FOR IRRIGATION

BY J. E. CHRISTIANSEN,¹ ASSOC. M. AM. SOC. C. E.

SYNOPSIS

Portable systems for sprinkling agricultural crops were first used in the Sacramento Valley of California in 1931, although in 1930 there were a few scattered installations in the southern part of the state. Sprinkling with stationary systems had been confined largely to citrus orchards and truck crops because of the relatively high investment required. Low-cost portable systems, however, made sprinkling a feasible method of irrigating large acreages of field crops such as beans, peas, onions, and sugar beets.

In 1932 the Division of Irrigation of the University of California began a study of sprinkling, principally to determine (1) the hydraulic characteristics of rotating sprinklers, (2) the loss of water by evaporation, (3) the hydraulic characteristics of sprinkler lines, (4) the cost of applying water by sprinkling, and (5) the general success of sprinkling as a method of irrigation. The scope of this paper is defined by the first three of these items.

PORTABLE SPRINKLER SYSTEMS FOR FIELD CROPS

In brief, a portable sprinkler system is a line of light-weight pipe with quick-acting couplings (on which rotating sprinklers are mounted) together with a pumping plant. Sprinkler pipe comes in standard lengths of 20 ft; other lengths are furnished on special order. Sprinklers are usually spaced 40 ft apart; but sometimes the spacing is 20 ft (or 30 ft when 30-ft pipe is used). The pipe is moved across the field, usually by two men who carry one length at a time. Several different makes of systems are now available, each with its own kind of coupling.² Formerly, only 4-in. pipe was available; but now one can obtain some of the systems in 1½-in., 2-in., 3-in., 4-in., 5-in., and 6-in. sizes and in any

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by May 15, 1941.

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² "Irrigation by Sprinkling," by J. E. Christiansen, *Journal, Am. Soc. of Agri. Engrs.*, December, 1937, p. 533, Fig. 1.

desired pipe length. Special quick-acting couplings are also made for use on smaller pipe for orchard systems.

As a rule, the systems operate with a portable pumping plant that pumps from a field ditch along one side of the field, or through the center. The plant shown in Fig. 1 is operating on two quarter-mile lines of 4-in. and 5-in. pipe,



FIG. 1.—TYPICAL PORTABLE PUMPING PLANT



FIG. 2.—MOVABLE SPRINKLER MACHINE PUMPING 500 GAL PER MIN FROM FIELD DITCH

with 61 sprinklers. Its capacity is approximately 1,100 gal per min. The most economical arrangement is to have the supply ditch through the center of the field with a sprinkler line extending in both directions. This minimizes the friction loss in the pipe line and necessitates shutting down only one line at a time when moving pipe, thereby appreciably increasing the operating efficiency of the system.

If the water supply is obtained from wells or if open ditches through the field are not feasible, a stationary pump and a pressure supply line are used. The latter may be either a portable pipe similar to the sprinkler line, or a stationary pipe with valve outlets. On stationary lines the valves are usually spaced about three times the distance the pipe is moved, so that three settings are made from each valve.

Another type of low-pressure system, consisting of light-weight perforated pipe, has recently come into use for field crops and pasture. Water is distributed over a strip 25 to 50 ft wide. These systems operate satisfactorily on pressures as low as 5 or 6 lb per sq in., but the effective width covered increases with the pressure up to about 25 lb per sq in. These systems apply the water at a relatively high rate, which limits their use to pervious soils and necessitates moving the pipe frequently.

Rotating nozzle lines are also used to some extent for irrigating truck crops and nurseries. They are probably more common in Eastern United States than in California.

One innovation is a movable sprinkler machine (Fig. 2) that pumps water from a field ditch and distributes it through two large part-circle sprinklers while traveling very slowly but continuously along the ditch. Several such machines³ were constructed in 1938. They have capacities up to 600 gal per min and effectively cover a strip about 250 ft wide when operating at a pressure of about 60 lb per sq in.

ORCHARD SPRINKLING SYSTEMS

The several types of orchard sprinkling systems may be divided into two general groups: The overhead systems with sprinklers mounted above the trees; and the under-tree systems with sprinklers mounted on short risers so that they distribute the water under the branches of the trees.

There are three types of overhead sprinkler systems: (1) Those with stationary sprinklers mounted above the trees; (2) those with portable sprinklers and risers that are moved from place to place on a stationary pipe system; and (3) those using portable pipe and sprinklers. There are also different types of under-tree systems, some using stationary sprinklers, others using portable sprinklers attached to hose or mounted on portable pipe. Stationary supply lines with valve outlets are generally used for portable systems.

The under-tree systems are gaining in popularity; some stationary overhead systems are even being converted into them. Under-tree sprinklers are usually spaced the same distance apart as the trees, so that each sprinkler covers an area

³ "Our New Movable Sprinkler Machines," by J. E. Christiansen, *Pacific Rural Press*, October 15, 1938.

equal to that occupied by one tree, whereas stationary overhead sprinklers are generally spaced three to four tree rows apart each way. Consequently the distribution of water is much better for the under-tree systems, since each tree is supplied with essentially the same quantity of water, although the distribution over the area occupied by each tree may not be uniform. Very little is gained by wetting the leaves, and often this practice is detrimental in that it spreads disease. Small under-tree sprinklers operate satisfactorily on pressures of 10 to 20 lb per sq in., whereas the larger sprinklers on overhead systems require 30 to 50 lb per sq in. The low pressures can sometimes be supplied by gravity, whereas pumping plants would be required for the higher pressures.

DISTRIBUTION OF WATER FROM SPRINKLERS

In studying the hydraulic characteristics of sprinklers, more than 300 tests were made at Davis, Calif., to determine the distribution of water from sprinklers. Most of these tests were for periods of one hour, some for $\frac{1}{2}$ hour. For most of the experiments, cans were spaced 10 ft apart (or closer) in both directions over the entire area covered by the sprinkler; and, for certain tests, as many as 280 cans were used. The water was measured to the nearest cubic centimeter (equivalent to 0.005 in. in depth) in a graduated cylinder. During most of the tests, wet-and-dry bulb temperatures were taken for use in evaporation studies. To measure the wind velocities, a four-cup anemometer was mounted in an exposed position about 10 ft above the ground. By means of a return-pressure line from the base of the sprinkler, with a pressure gage mounted near the control valve, a nearly constant pressure was maintained. The plotting of the tests is illustrated in Fig. 3, which shows the distribution obtained with one sprinkler under favorable conditions (sufficient pressure, no wind, and a fairly slow and uniform rotation).

The effect of low pressure is shown in Fig. 4. With a pressure of 20 lb per sq in. most of the water was thrown to the outside edges of the area covered, with very little falling between 10 and 20 ft from the sprinkler. Fig. 5 shows the effect of wind. The area covered is offset about 2 ft for each mile per hour of wind, with a high concentration near the sprinkler, especially on the windward side.

A cause of uneven distribution is often a variable rate of rotation of the sprinklers. Every sprinkler tested varied somewhat in this respect, some much more than others. In one case (Fig. 6), the angular velocity was eight times as great when the main nozzle was discharging to the east as when it was discharging to the southwest. For a study of rotation, an anemometer recorder was converted into a rotation recorder by gearing it so that the drum rotated at a speed of one inch in 50 sec. Attached to the sprinkler was a commutator arrangement that closed an electrical circuit every 30° or 60°, as desired. In this way the actual rate of rotation for a large number of revolutions was recorded automatically. Uneven rotation is not caused by wind, as is commonly supposed.

The rate of rotation greatly affects the distribution of water. For maximum coverage, a sprinkler should rotate less than 1 rpm. Often it may rotate as

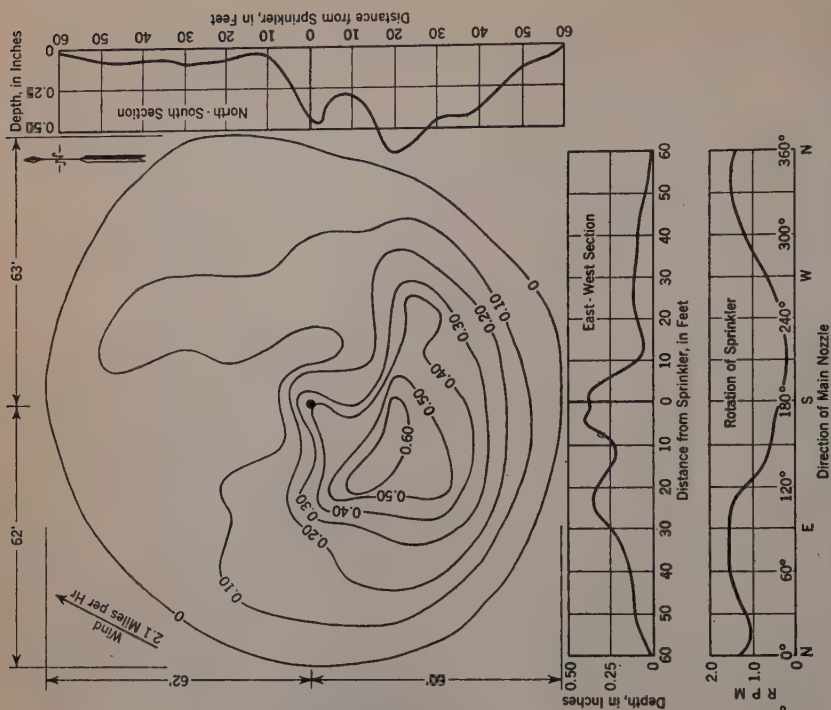


FIG. 6.—TEST 103. DISTRIBUTION OF WATER FROM SPRINKLER WITH VARIABLE RATE OF ROTATION; PRESSURE, 40 LB PER SQ IN.

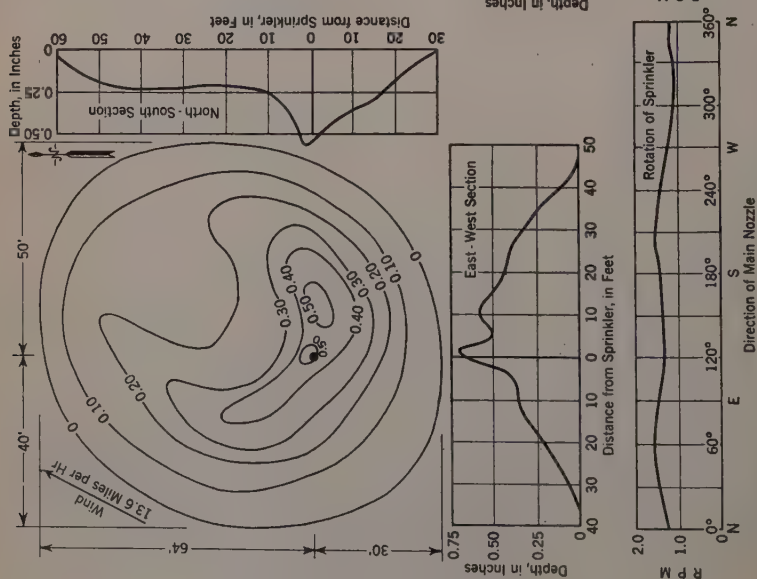


FIG. 5.—TEST 112. EFFECT OF WIND (13.6 MILES PER HR) ON DISTRIBUTION OF WATER FROM SPRINKLER WITH $\frac{1}{2}$ -IN. MAIN NOZZLE. PRESSURE, 40 LB; ROTATION, 1.3 RPM

slowly as $\frac{1}{2}$ rpm. The slower the rotation, the larger the area covered. When improperly adjusted and allowed to rotate at 20 rpm or more, sprinklers cover only about half the area they would cover at a slow rate under the same pressure. In one instance, a sprinkler was observed to be rotating very rapidly, and a count showed the speed to be about 90 rpm. The effect of rapid rotation is shown in Fig. 7, in which case a speed of 13.6 rpm reduced the area covered to about 55% of what it would have been at 1 rpm. Unless such sprinklers are mounted closer together, the uniformity of application will be poor. In Fig. 7

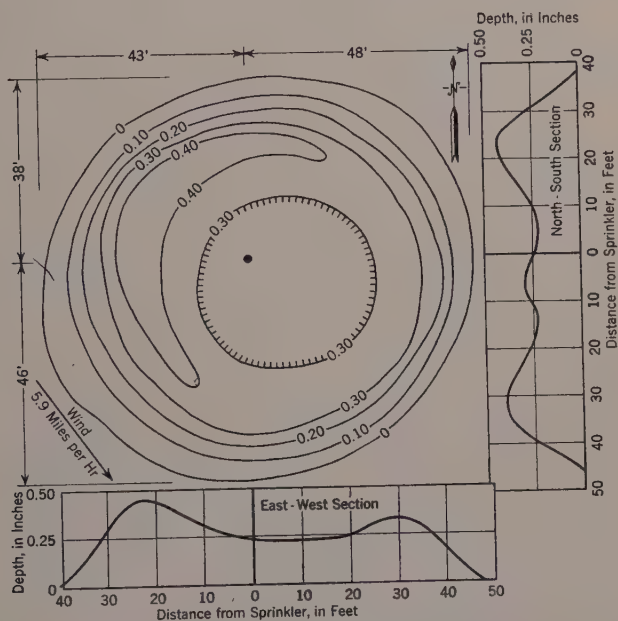


FIG. 7.—TEST 21. DISTRIBUTION OF WATER FROM SPRINKLER ROTATING AT A SPEED OF 13.6 RPM

the nozzles were $\frac{9}{32}$ in. and $\frac{5}{32}$ in., and the pressure was 40 lb per sq in. Under similar conditions, but with rotation less than 1 rpm, the diameter covered would have been about 115 to 120 ft.

Patterns showing the distribution of water from individual sprinklers can be used to determine the uniformity of distribution from a group of sprinklers spaced any distance apart. The resulting distribution from sprinklers with patterns corresponding to Fig. 3, for the customary spacing of 40 ft by 50 ft, is shown in Fig. 8(a). The variation is from about 0.7 in. to slightly more than one inch. Fig. 8(b) shows the resulting distribution for patterns corresponding to Fig. 4, illustrating the effect of low pressure. Here the variation is from about 0.2 in. to more than one inch. Fig. 8(a) illustrates the uniformity obtained under favorable conditions and Fig. 8(b) illustrates what occurs when pressures are inadequate.

To compare sprinkler patterns and to determine how various spacings affect the resulting distribution of water, one needs a numerical expression that can serve as an index of the uniformity secured. For this purpose an expression called the uniformity coefficient was adopted. The uniformity coefficient (C_u) expressed as a percentage is defined by the equation

$$C_u = 100 \left(1.0 - \frac{\sum x}{Mn} \right) \dots \dots \dots (1)$$

in which d is the deviation of individual observations from the mean value m , and n is the number of observations. When the intensity of application at any number of equally spaced points over the entire area covered by a sprinkler is determined, the uniformity coefficient can be computed for any spacing (in either direction) which is a multiple of the spacing of the points of observation. Thus a complete analysis of one sprinkler pattern to determine the best spacing and the resulting uniformity of application involves numerous computations.

So that all the tests might be analyzed, the problem of determining the proper spacing for best distribution was simplified by calculating the uniformity coefficients for a close spacing in one direction by various spacings in the other direction. Most of the tests were analyzed for spacings of 5 or 10 ft in one direction by 40 to 100 ft in the other direction, and some were also analyzed for spacings of 40 ft in one direction by 40 to 100 ft in the other direction. The analyses indicate that the distribution is usually satisfactory when the spacing along the line is not more than half the maximum satisfactory spacing between lines as indicated by the uniformity coefficient.

Before discussing the uniformity coefficients obtained from the tests, one might well consider some symmetrical patterns of various geometric shapes

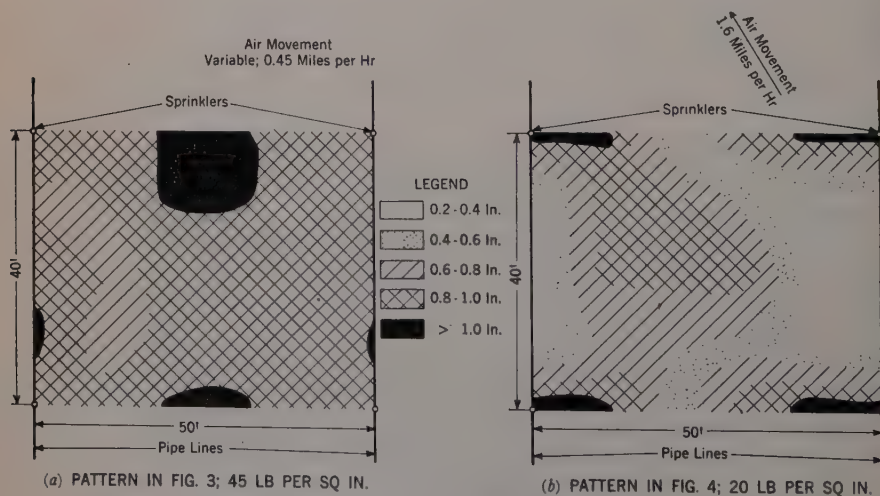


FIG. 8.—DISTRIBUTION RESULTING FROM SPRINKLERS HAVING VARIOUS PATTERNS WHEN SPACED 40 FT BY 50 FT APART

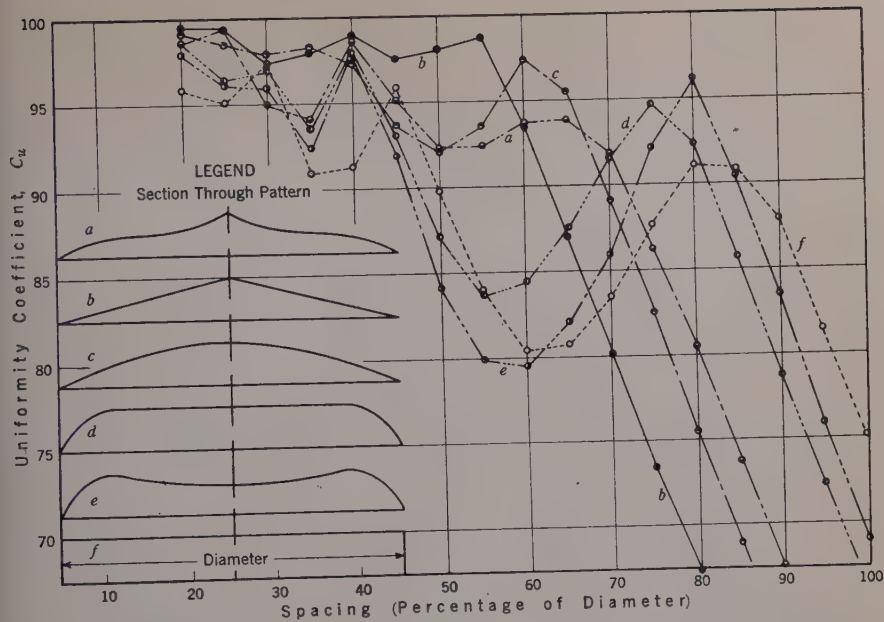


FIG. 9.—COEFFICIENTS OF UNIFORMITY FOR DIFFERENT SPACINGS OF GEOMETRICAL PATTERNS

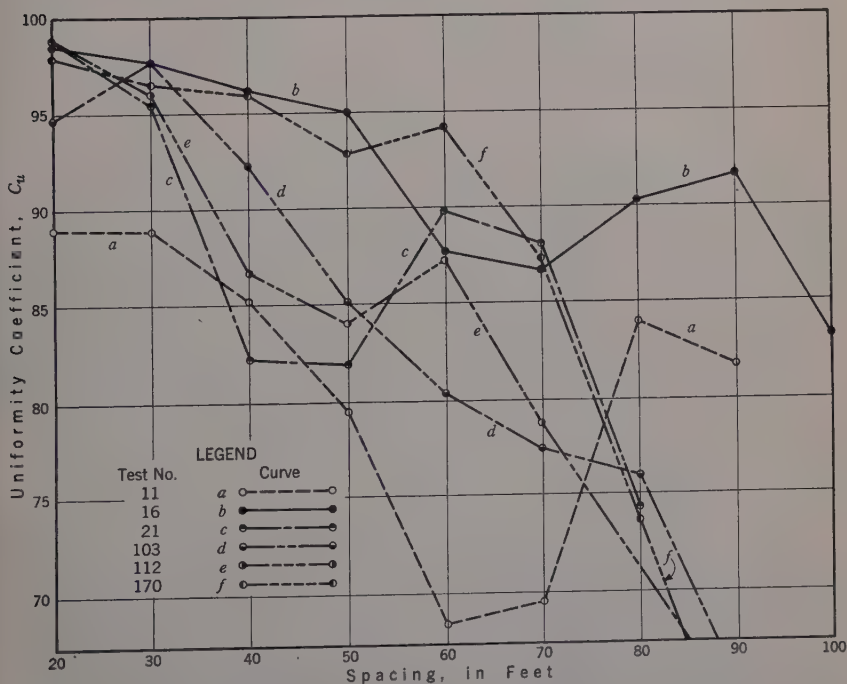


FIG. 10.—UNIFORMITY COEFFICIENTS FOR THE SPRINKLER PATTERNS SHOWN IN FIGS 3 TO 7

approximating those of actual sprinkler patterns. The results of such a study are shown in Fig. 9. These patterns fall in two general groups: *a*, *b*, and *c*, which taper gradually to the edges; and *d*, *e*, and *f*, with more abrupt edges. The first three patterns have the highest uniformity coefficients for all spacings up to about 65% of the diameter covered, whereas the last three have fairly high coefficients for spacings 70% to 85% of the diameter, but very low coefficients for spacings between 45% and 70%. This characteristic indicates excessive overlap. Pattern *b*, Fig. 9, has the most consistently high coefficient for all spacings up to 55% of the diameter.

As this analysis shows, the most uniform application can be obtained with a conically shaped pattern, provided the sprinklers are spaced not more than about 55% of the actual diameter covered. Good results, apparently, can be obtained with patterns similar to *d* or *e* (Fig. 9), if the sprinklers are spaced correctly. Actually, however, the diameter of the area covered varies greatly with the wind, the speed of rotation, and, to some extent, the pressure, so that it is difficult to space sprinkler lines properly to obtain good distribution with patterns of this type.

A similar analysis was made of all the sprinkler tests for which complete data were obtained. Fig. 10 shows the variation in the average values of the uniformity coefficient for different spacings for the tests shown in Figs. 3 to 7.

Judging from this analysis, together with field observations, sprinklers must be placed closer together than is indicated by the study of symmetrical patterns. The usual spacings of 20 ft, 30 ft, and 40 ft along the line and 50 ft to 70 ft between lines give fair results when there is adequate pressure and little wind. The customary spacings of 60 ft to 90 ft both ways for stationary overhead orchard systems, however, cause an appreciable variation in application.

EVAPORATION LOSSES

From the quantities of water caught in the cans the total quantity of water falling on the area was calculated and compared with the discharge from the sprinklers. These data were expected to give some idea of the quantity evaporated from the spray. Although the method proved quite accurate for determining the total evaporation loss, nearly all the loss was found to occur from the cans before the water was measured; very little evaporated before the water reached the ground. This conclusion was reached from a consideration of the energy required to evaporate water and from tests to determine the change in temperature of the water from the time it leaves the nozzle until it reaches the ground. An approximate expression giving the portion of water lost from the spray is

$$E = \frac{c (t_1 - t_2)}{L} \left[\frac{p_w - p_a}{p_w - p_a - 0.00037 B (t_a - t_w)} \right] \dots \dots \dots (2)$$

in which E = the portion of water loss by evaporation from the spray; c = the specific heat of water, calories per gram per degree Fahrenheit; L = the latent heat of vaporization, calories per gram; t_1 = the temperature of the water at the nozzle, in degrees Fahrenheit; t_2 = the temperature of the water striking the ground, in degrees Fahrenheit; t_w = the mean water temperature, in degrees

Fahrenheit; t_a = the air temperature, in degrees Fahrenheit; p_w = the vapor pressure of water at temperature t_w , in inches of mercury; p_a = the pressure of the water vapor in the air, inches of mercury; and B = the barometric pressure, in inches of mercury.

Evaluating the constants, and letting $c = 0.555$, $L = 585$, and $B = 30$, Eq. 2 becomes

$$E = 0.00095 (t_1 - t_2) \left[\frac{p_w - p_a}{p_w - p_a - 0.011 (t_a - t_w)} \right] \dots \dots \dots (3)$$

One can derive a similar expression from the general evaporation formula,⁴

$$E = \frac{I - H_s - k - R_b}{L (1 + r)} \dots \dots \dots (4)$$

by assuming that the insolation I , the conduction k , and the back radiation R_b are negligible and can be omitted. The sensible heat, H_s , is indicated by the change in the temperature of the water. The other terms are as follows: L , the heat of vaporization; and r , the ratio of energy transfer by convection to that by evaporation (called Bowen's ratio), which is a function of the vapor-pressure deficit and of the difference in air and water temperature.

Several determinations of the temperature drop in the water between the sprinkler and the ground were made on field installations and at Davis. The evaporation losses for these tests, computed from Eq. 4, did not exceed one per cent. The drop in temperature, and therefore the evaporation loss, depends largely upon the size of drops, which in turn depends upon pressure, wind, and type of sprinkler. Apparently, however, the loss from the spray is very little, and the principal loss by evaporation is from the wet surface of soil and plants during and following the application of water.

The rate of this loss probably exceeds that from a free water surface because of the relatively larger area exposed. The rate decreases rapidly, however, as the surfaces dry, and becomes negligible after about a week, during which time the total loss may have exceeded an inch of water. In the interior valleys of California, rates of evaporation from standard Weather Bureau pans often exceed 0.40 in. per day; hourly rates frequently exceed 0.05 in., and average about 0.04 in. during the afternoon. The total evaporation loss frequently amounts to a large part of the application, especially when frequent light applications are made.

HYDRAULICS OF SPRINKLER LINES

One interesting problem in connection with sprinkler systems concerns the hydraulic losses at branch outlets along sprinkler lines. All the water does not flow through the entire line; it is distributed through side outlets equally spaced along the line. At each of these outlets there is a decrease in mean velocity in the pipe (assuming no change in pipe size) accompanied by an increase in pressure due to conversion of velocity head to pressure. There is also an hydraulic loss between the main and the sprinkler outlets. Consider a typical

⁴"Evaporation as a Function of Insolation," by Burt Richardson, *Transactions, Am. Soc. C. E.*, Vol. 95 (1931), pp. 996-1019.

outlet along the line, sections 1, 2, and 3, representing, respectively, sections upstream and downstream from the outlet and in the side outlet. From Bernoulli's theorem, assuming no energy loss, one can show that

$$p_2 - p_1 = \frac{w v_1^2}{2g} (2 r_q - r_q^2) \dots \dots \dots (5)$$

In Eq. 5, p_2 and p_1 are the pressure intensities on sections 2 and 1, respectively, v_1 = the mean velocity at section 1; g = acceleration due to gravity; r_q = ratio of $\frac{Q_3}{Q_1}$, or the proportion of water leaving the side outlet; and w = the weight of a cubic foot of water. John A. Oakey,⁵ Assoc. M. Am. Soc. C. E., making a different assumption, derives an equation that can be written

$$p_2 - p_1 = \frac{w v_1^2}{2g} (3 r_q - r_q^2) \dots \dots \dots (6)$$

His experimental data, however, show that for values of r_q less than 0.7 the actual increase in pressure was approximately that given by Eq. 5. The energy loss between sections 1 and 3 can be determined only by experiment. Professor Oakey⁵ gives values of K_3 from which the loss can be determined from the expression

$$e_1 - e_3 = \left(1 - r_q + \frac{K_3 r_q^2}{r_p^2} \right) \frac{v_1^2}{2g} = \frac{v_1^2}{2g} (1 - r_q) + \frac{K_3 v_3^2}{2g} \dots \dots (7)$$

in which e = energy; r_p is the ratio of the pipe area at section 3 to the pipe area at section 1; and K_3 is an empirical coefficient having values dependent both upon r_p and r_q . For small values of r_p , apparently, the value of the coefficient K_3 would be more nearly constant if the expression were written

$$e_1 - e_3 = \frac{v_1^2}{2g} + \frac{K_3 v_3^2}{2g} \dots \dots \dots (8)$$

Professor Oakey's data show that for $r_p = 0.0557$, K_3 varied from 0.53 for $r_q = 0.1$ to 0.36 for $r_q = 1.0$. Since values of v_3 are fairly constant along a sprinkler line and seldom exceed 8 ft per sec, the last term in Eq. 8 will seldom exceed 0.5 ft head. The conclusion reached is that the velocity in the sprinkler line has little effect on the flow in the outlets, and that the energy loss between sections 1 and 3 is of small consequence because, in comparison with the pressure required, it is appreciable only near the upper end of the line where the pressure is excessive.

Field tests were made to determine the friction factors or coefficients for the special sprinkler pipe and couplings used for portable sprinkler systems. These tests presented some difficulty because pipe-line velocities were sometimes as high as 15 ft per sec with pressures of 60 lb or more per sq in. A transverse pitot tube for measuring the flow did not prove satisfactory. The method finally adopted for these tests consisted of calibrating the sprinklers to determine

⁵ "Hydraulic Losses in Short Tubes Determined by Experiments," by John A. Oakey, *Engineering News-Record*, June 1, 1933, pp. 717-718.

the relation between pressure and discharge for all nozzle combinations and then to determine the pressure along the line by means of a pressure gage and a special fitting, slipped over the main nozzle of the sprinkler. It was necessary to correct for the increase in pressure caused by shutting off the flow of the nozzle being tested. As tests show, this correction was approximately constant along a line; one could determine it readily by noting the increase in pressure on another gage installed on the line or simply by momentarily stopping the flow of another nozzle while the gage was being used on one of the sprinklers. A pressure reading was taken on each sprinkler on a line from which the sprinkler discharge and the total discharge were determined from the calibration. The pressures were plotted on a large sheet of coordinate paper, and a smooth curve was drawn through the points. The pressure drop for each space between sprinklers was then taken from the curve and plotted against the discharge on logarithmic paper. The curve was drawn through these points, showing the relation between flow and pressure drop along the line. An interesting fact about these curves was that the best-fitting straight line showed the pressure drop to be a function of the square of the velocity in the pipe, instead of some lower power as indicated by most of the empirical formulas. For this reason the Weisbach formula,

$$h_f = \frac{f l v^2}{2 g D} \dots \dots \dots (9)$$

was adopted, and the average value of f for 4-in. pipe was found to be about 0.018, there being no appreciable difference between the different makes of pipe tested. (In Eq. 9, h_f = head loss due to friction; f = coefficient of friction; l = length of pipe; and D = diameter of pipe.)

However, when the values of f were computed from the pressure drops in each section of pipe between sprinklers, corrected by adding the pressure rise due to the decrease in velocity at the sprinklers to obtain the true friction loss, the values of f obtained were not constant but showed the usual form of curve when plotted against Reynolds' number.

In sprinkler problems one usually thinks of discharge in terms of gallons per minute and friction losses in pounds per square inch. For convenience the Weisbach formula can be written

$$p_f = \frac{0.0134 f l Q_o^2}{D_i^5} \dots \dots \dots (10)$$

in which p_f = the friction loss in pounds per square inch, l = the length of the line in feet, Q_o = the flow in gallons per minute, and D_i = the inside diameter of the pipe in inches.

A difficult task in the design of sprinkler systems is to determine pressure losses along a line with a large number of sprinklers. Although one can compute the loss between each two adjacent sprinklers, for long lines with twenty or more sprinklers this procedure becomes wearisome. Furthermore, in such computations one must begin at the distal end of the line and work back toward the source, beginning with an assumed pressure. A precise algebraic expression for the pressure loss in a long line of pipe with multiple outlets is very

involved and therefore is of little practical value. Approximate expressions, however, from which the pressure loss can be determined quite accurately for limited conditions are very useful. If one assumes that the sprinklers are spaced s feet apart and that each discharges the same quantity of water (q),

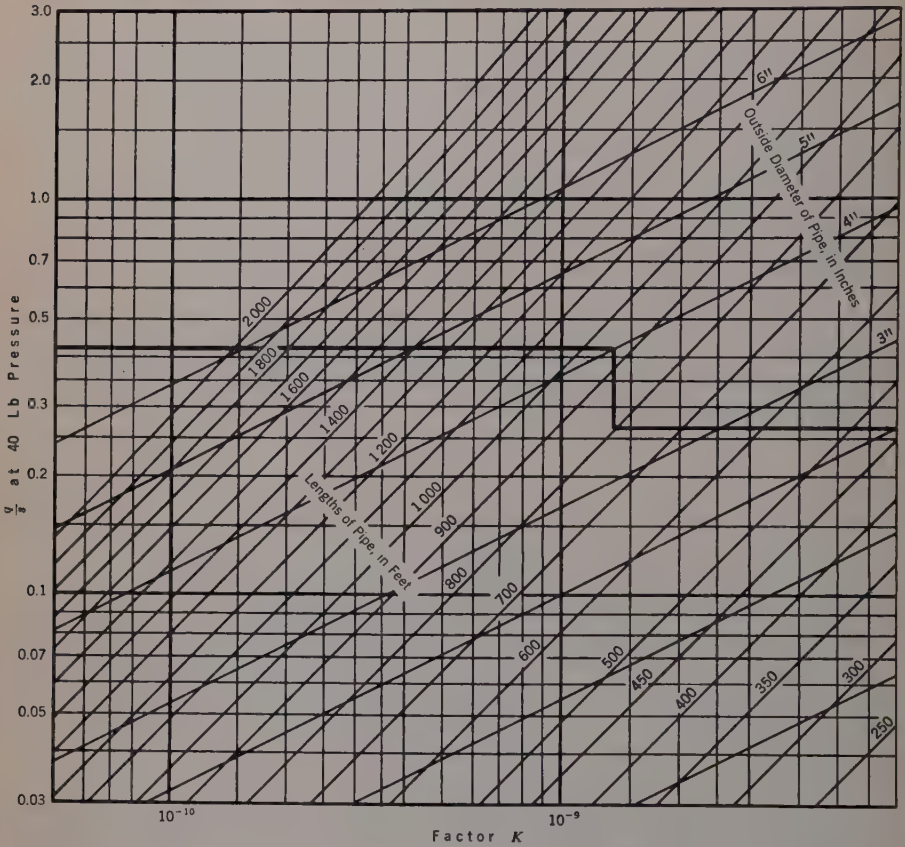


FIG. 11.—LOGARITHMIC DIAGRAM FOR DETERMINING

the total pressure loss for a line with N sprinklers and N spaces would be

$$p_f = \frac{0.0134 f s q^2 \Sigma N^2}{D_i^5} \dots \dots \dots (11)$$

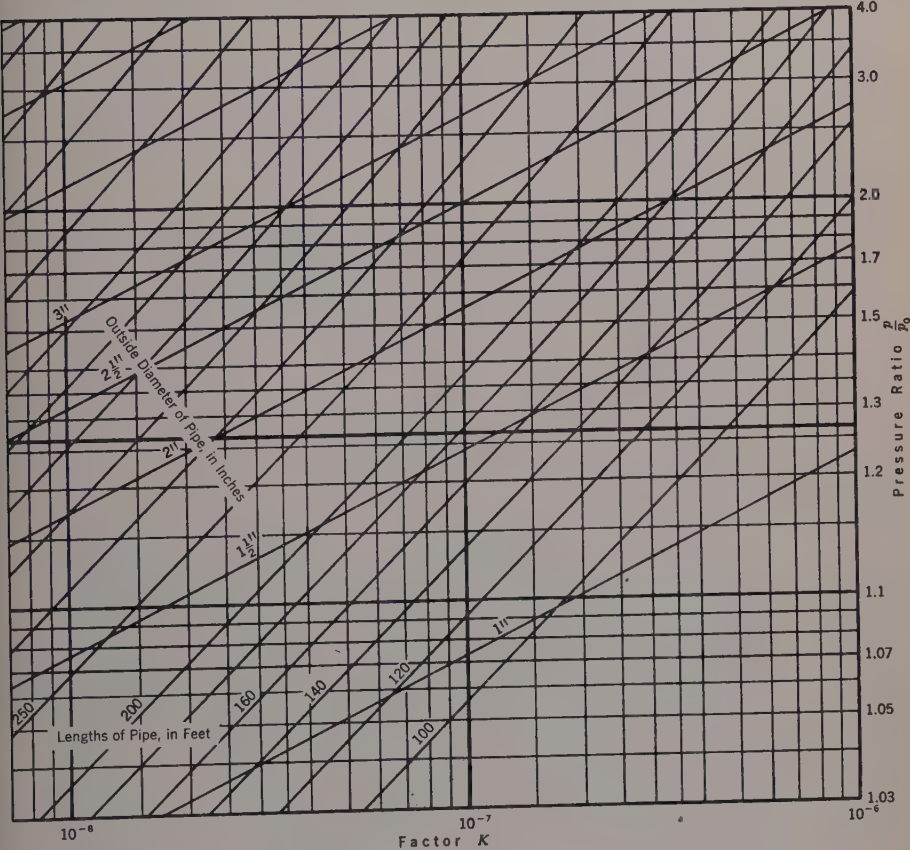
Because of the friction loss along a line, however, the sprinklers do not discharge the same quantity of water, even if the same nozzle sizes are used throughout. If the total pressure loss is relatively small, as compared with the pressure on the line, and if the computations are based on the average discharge of the sprinklers, the results will check very closely with those determined by detailed computations. The value of ΣN^2 can be determined from the expression

$$\Sigma N^2 = \frac{N^3}{3} + \frac{N^2}{2} + \frac{N}{6} \dots \dots \dots (12)$$

Eq. 11 can be written

$$p_f = \frac{0.0134 f L Q^2 (\Sigma N^2)}{D_i^5 N^3} \dots \dots \dots (13)$$

in which the first part of Eq. 13 would be the friction loss if all the water flowed



PRESSURE LOSSES IN PORTABLE SPRINKLER LINES

through to the end of the line. From Eq. 12,

$$\frac{\Sigma N^2}{N^3} = \frac{1}{3} + \frac{1}{2 N} + \frac{1}{6 N^2} \dots \dots \dots (14)$$

The simplest method of obtaining friction loss along a sprinkler line, therefore, is first to compute the loss, assuming that all the water flows through the entire line, and then to multiply this by a factor that depends only on the number of sprinklers. For this purpose any friction-loss table or graph applying to the pipe can be used.

The friction loss along a sprinkler line is directly proportional to the pressure at any point on the line. It has been found convenient to think in terms

of pressure ratio $\frac{p}{p_o}$, the ratio of the pressure at any point on the line to the pressure at the distal end. This ratio is constant at a given point on a line regardless of the operating pressure at the pump. Assuming that all sprinklers have the same nozzle sizes, it follows that the discharge ratio (the discharge of any sprinkler on the line to the discharge of the sprinkler at the distal end) is equal to the square root of the pressure ratio. Thus $\frac{q}{q_o} = \sqrt{\frac{p}{p_o}}$

$= 1 + 0.5 \left(\frac{p}{p_o} - 1 \right)$ approximately. The average sprinkler discharge is approximately

$$q_a = q_o \left[1 + 0.12 \left(\frac{p_n}{p_o} - 1 \right) \right] \dots \dots \dots (15)$$

in which p_n is the pressure at the sprinkler nearest the pump. The total discharge of the line is

$$Q = N q_a \dots \dots \dots (16a)$$

and the horsepower requirement,

$$P_H = \frac{Q p_p}{1,715 \eta} \dots \dots \dots (16b)$$

in which η is the efficiency of the pump and drive, and p_p is the pressure corresponding to the total pumping head.

Fig. 11 is a logarithmic chart which enables the engineer to determine quickly the pressure ratios for different pipe sizes and lengths. Since the tests on friction loss covered only 4-in. and 5-in. pipe, this graph is based on friction factors as given by R. J. S. Pigott⁶ for new, smooth steel pipe and for Reynolds' numbers corresponding to velocities of $6\frac{1}{2}$ ft per sec. The values of f used for the different pipe sizes were as follows: 1-in., 0.028; $1\frac{1}{2}$ -in., 0.025; $2\frac{1}{2}$ -in., 0.022; 3-in., 0.021; 4-in., 0.020; 5-in., 0.019; and 6-in., 0.018. These values, being higher than those from tests on the 4-in. pipe, are believed safe for design purposes. To incorporate all factors on a single graph, one can plot values of $\frac{q}{s}$ at a constant pressure (a pressure of 40 lb per sq in. being used) against a parameter K , and then plot K against the length of the line, l , to obtain the pressure ratio $\frac{p}{p_o}$, in which p is a pressure at any distance, l , from the distal end of the line.

To use the graph one must first determine the discharge of the sprinkler at a pressure of 40 lb per sq in. If the discharge of the sprinkler at any pressure is known, the discharge at 40 lb per sq. in. can be determined readily, since the discharge of the sprinkler is proportional to the square root of the pressure. For example, to find the friction loss in 800 ft of 4-in. pipe with sprinklers discharging 19 gal per min at 50 lb of pressure, spaced 40 ft apart, one must first determine the discharge at 40 lb of pressure. Thus, $q_{40} = 19 \sqrt{\frac{40}{50}}$

⁶"The Flow of Fluids in Closed Conduits," by R. J. S. Pigott, *Mechanical Engineering*, August, 1933, pp. 497-501, and 515.

= 17. With sprinklers spaced 40 ft apart, $\frac{q_{40}}{s} = \frac{17}{40} = 0.425$. Entering the left side of the diagram with this value, follow to the right to the intersection with 4-in. pipe. Then drop vertically downward to the intersection with the 800-ft line, and read the pressure ratio $\frac{p}{p_o} = 1.27$ on the right margin. If the pressure at the distal end, p_o , equals 30 lb per sq in., the pressure 800 ft from the end would be $1.27 p_o = 38.1$ lb; the pressure loss would be 8.1 lb. This diagram has proved very convenient for determining the proper pipe size under a given set of conditions or for determining the permissible length of pipe of each size in a combination line having two or more pipe sizes.

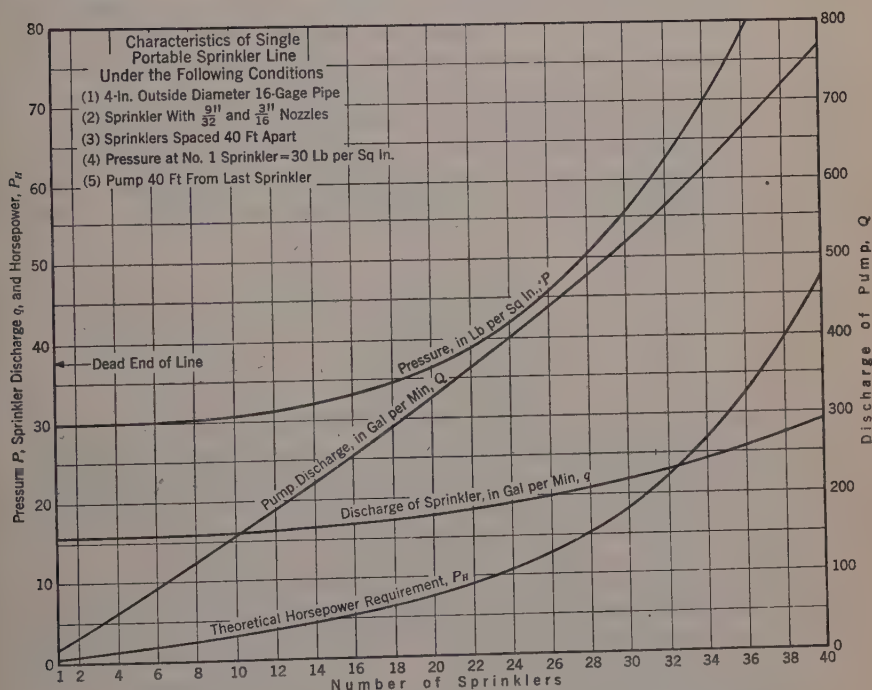


FIG. 12.—HYDRAULIC CHARACTERISTICS OF A PORTABLE SPRINKLER LINE

In designing a sprinkler system it is customary to start with an assumed value of p_o equal to the minimum pressure for which satisfactory sprinkler performance can be obtained. The dimensions of the field govern the length of the line; and since farm tractors required for other farm work are generally used as a source of power, the available power is generally known in advance. One must then ascertain the sizes of the sprinkler nozzles, the pipe, and the pump required.

When no provision is made for regulating the discharge of sprinklers along the line, it is a good practice to proportion the pipe sizes and nozzle sizes so that the over-all pressure ratio does not exceed 1.2. This arrangement limits

the discharge ratio to approximately 1.1, permitting a 10% variation in sprinkler discharge along the line. On large systems it may not be advisable to limit the ratio to this value because pipe larger than 4 in. in diameter is heavy and difficult to carry. Many systems in use have pressure ratios in excess of 2; one tested had a ratio of approximately 3. Needless to say, such systems are very inefficient, and the pressure at the distal end of the line is usually far too low for satisfactory sprinkler performance. Lately, on one make of sprinkler system, the risers are provided with a shut-off cock and a pressure-valve arrangement so that the pressure on all sprinklers on the line can be determined readily and equalized.

Fig. 12 shows the hydraulic characteristics of a portable sprinkler line under the following conditions: (a) 16-gage pipe, outside diameter 4 in.; (b) sprinklers with $\frac{9}{32}$ -in. and $\frac{3}{16}$ -in. nozzles; (c) spacing of sprinklers, 40 ft; (d) pressure at the No. 1 sprinkler, 30 lb per sq in.; and (e) distance from pump to last sprinkler, 40 ft. The actual horsepower requirement is determined by dividing the theoretical requirement by the efficiency of the pump and drive. Efficiencies of 50% to 70% are assumed for this purpose. This graph emphasizes the advantages of using two short lines supplied from a source through the center of the field instead of a single line of twice that length, supplied from a source along one side of the field.

APPENDIX

NOTATION

The following notation conforms essentially with American Standard Symbols for Hydraulics compiled by a Committee of the American Standards Association⁷ with Society representation and approved by the Association in 1929:

A = area:

A_1 = area of section 1;

B = barometric pressure;

C = coefficient:

C_u = uniformity coefficient;

c = specific heat of water;

D = diameter:

D_i = inside diameter of pipe in inches;

E = evaporation; the portion of water loss by evaporation;

e = energy;

f = friction coefficient;

g = gravity constant;

H = heat:

H_s = sensible heat;

h = hydraulic head:

h_f = head loss due to friction;

⁷ A. S. A.—Z 10b—1929.

I = insolation;

K = an empirical coefficient:

K_3 = an empirical coefficient having values dependent on both r_p and r_q ;

k = thermal conductivity; conduction;

L = latent heat of vaporization;

l = length;

M = mean value;

m = mass;

N = the total number of;

n = any number of observations;

P = power:

P_H = horsepower requirement;

p = pressure intensity; pressure loss:

p_a = pressure of water vapor in the air;

p_f = pressure loss due to the pipe friction;

p_n = pressure intensity at the sprinkler nearest the pump;

p_o = pressure intensity at the distal end of a pipe;

p_p = the pressure corresponding to the total pumping head;

p_w = vapor pressure of water at t_w ;

p_1 = pressure intensity at section 1;

Q = quantity, or rate of flow:

Q_g = flow in gallons per minute;

q = rate of flow through single sprinklers:

q_a = average discharge of the sprinklers on a line;

q_o = flow through a sprinkler at the distal end of pipe;

R = radiation:

R_b = back radiation;

r = ratio of energy transfer by convection to energy transfer by evaporation (called Bowen's ratio):

r_p = ratio of the pipe area at Point 3 to the pipe area at Point 1;

$r_q = \frac{Q_3}{Q_1}$, or the proportion of water leaving the side outlet;

s = spacing of sprinklers;

t = temperature:

t_a = air temperature;

t_w = the mean water temperature;

t_1 = temperature of water at the nozzle;

t_2 = temperature of water as it strikes the ground;

v = velocity:

v_1 = velocity at section 1;

w = weight of a cubic foot of water;

x = deviation from the mean;

η = efficiency.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

AN INVESTIGATION OF STEEL RIGID FRAMES

Discussion

By C. J. POSEY, ASSOC. M. AM. SOC. C. E.

C. J. POSEY,¹¹ Assoc. M. Am. Soc. C. E. (by letter).^{11a}—When continuous corners of structural steel first came into use not many years ago, their design usually was based upon the methods that had proved satisfactory for the design of beams and girders. Structurally, however, a corner is very different from a beam, although it may be built with the same cross-sectional shape. The fact that serious consequences may result from designing a corner by the methods used for designing a beam is gradually becoming known. The authors' tests and the companion investigation at the National Bureau of Standards reveal some of the basic differences in structural action of the two types of members.

The most important structural characteristic of a steel corner of I-shaped cross section having a curved flange is the tendency of the web to buckle.¹² When the inside of the corner is in compression (as was the case with the frames tested by the authors) the curved flange puts a large radial compressive loading on the web, and if the web is not adequately reinforced with stiffeners, it will buckle. The tendency to buckle appears at comparatively low loads.

The existence of such an effect is recognized in the report on the curved inner flange riveted steel rigid frame tested at the Bureau of Standards. Messrs. Stang, Greenspan, and Osgood include the following statement among their conclusions:

"The design was such that the frame failed by elastic instability. The inner-flange angles at failure were stressed to less than half their yield point. This emphasizes the necessity of providing adequate bracing for the inner flange at the knees of rigid frames of this type."¹⁴

NOTE.—This paper by Inge Lyse, M. Am. Soc. C. E., and W. E. Black, Jun. Am. Soc. C. E., was published in November, 1940, *Proceedings*.

¹¹ Associate Engr., Iowa Inst. of Hydr. Research; Associate Prof., Hydraulics and Structural Eng., State Univ. of Iowa, Iowa City, Iowa.

^{11a} Received by the Secretary December 11, 1940.

¹² "Handling Corners in Rigid Frames," by B. J. Lambert, M. Am. Soc. C. E., and C. J. Posey, *Engineering News-Record*, August 4, 1938, p. 147.

¹⁴ "Strength of a Riveted Steel Rigid Frame Having a Curved Inner Flange," by Ambrose H. Stang, Martin Greenspan, and William R. Osgood, *Research Paper No. 1161, Journal of Research, National Bureau of Standards*, Vol. 21, 1938, p. 853.

What is vaguely referred to as "elastic instability" is nothing other than the tendency of the web to buckle due to the radial compressive forces. Bracing the inner flange, except by means of stiffeners, would not decrease the stress in the web.

The authors do not mention this important phenomenon at any point in their paper. Fig. 7 seems to show that sidewise deflection of the inner flange was prevented to some extent by the frame holding the specimen. Such support is seldom, if ever, provided when curved knees are used in construction. Even with this type of bracing, which decreases the effective slenderness ratio of the web and thus increases its resistance to buckling, the tendency toward "elastic instability" should become evident. The data included with the present paper do not seem to permit any comparison that would reveal this, although it may be noted that the span with curved flanges was more flexible, in comparison with computed values, than was the span with a sharp corner, although the fabrication of the latter was admittedly inferior.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

RELIABILITY OF STATION-YEAR RAINFALL-FREQUENCY DETERMINATIONS

Discussion

BY PAUL V. HODGES, M. AM. SOC. C. E.

PAUL V. HODGES,²³ M. AM. SOC. C. E. (by letter).^{23a}—In the determination of the probable maximum floods to be expected in different parts of the country, the pluvial indexes and their standard errors give a basis from which comparisons and computations can be made. This paper is of considerable assistance and value in the proper appraisal of records of storm rainfall over the eastern half of the United States.

In an unpublished study of the Grand River, in South Dakota, by the writer, to determine the probable maximum flood, for a spillway design, the pluvial index of Quadrangle 18-B was used (see Fig. 2). The pluvial indexes, in inches, for this quadrangle are given in Table 4A. The indication of the standard error as it affects the frequency in years is as follows:

Frequency, in years	Pluvial index (in inches)
15± 3.3.....	2.7
25± 7.0.....	4.2
50±19.9.....	4.4
100±55.9.....	4.8

The extremes of these periods were plotted against the pluvial index. By connecting the plotted points the range in the pluvial index for any frequency is obtained as follows:

Frequency, in years	Range in the pluvial index (in inches)
15.....	2.3 to 3.5
25.....	3.5 to 4.31
50.....	4.3 to 4.85
100.....	4.54 to 5.27

The errors of these various quantities are reflected in their plotting. With more Weather Bureau stations, well distributed, and with longer records, the

NOTE.—This paper by Katharine Clarke-Hafstad was published in November, 1940, *Proceedings*.

²³ Hydrographic Engr., U. S. Indian Irrig. Service, Denver, Colo.

^{23a} Received by the Secretary December 16, 1940.

points, no doubt, would plot on smooth curves and the results would be more consistent.

Maximum Rainfall to be Expected Once in 100 Years.—To determine the probable maximum flood to be expected on the Grand River in northwestern

TABLE 4A.—PLUVIAL INDEXES, IN INCHES

Duration of storm, in days	FREQUENCY, IN YEARS			
	15	25	50	100
1	2.0	2.5	3.4	4.0
2	2.7	3.3	4.1	4.2
3	2.7	4.0	4.3	4.5
4	2.7	4.2	4.4	4.8
5	2.7	4.2	4.4	4.8
6	2.7	4.2	4.4	4.8
% ^a	21.8	27.9	39.8	55.9

TABLE 4B.—INDEXES OF RAINFALL TO BE EXPECTED ONCE IN 100 YEARS

					TIME-AREA-DEPTH RELATION			
					At the point	4,085 sq miles	5,664 sq miles	6,000 sq miles
					4.50	3.20	3.03	3.00
					5.10	3.60	3.43	3.40
					5.50	4.10	3.93	3.90
					5.70	4.30	4.05	4.00
					5.80	4.40	4.15	4.10
					5.80	4.40	4.15	4.10
% ^a					* Standard error (percentages)			

South Dakota the "Hydrograph Method" was used and compared with the largest floods of nearby streams. The maximum rainfall to be expected once in 100 years for storms of different durations was derived for different areas. The pluvial index for quadrangles of the Grand River drainage area is given in *Technical Reports*, Part 5, of the Miami Conservancy District.⁴

The pluvial index gives the rainfall to be expected at any point. To determine what the depth over an area would be, a study was made of a number of storm areas and the time-area-depth relation was determined. Table 4B shows the depth of rainfall to be expected once in 100 years over different areas. The quantities given in this table were derived from the pluvial indexes of two quadrangles, 18-B and 18-C, in the northwestern part of South Dakota.

Flood Runoff, Grand River Near Wakpala, S. Dak.—The runoff of the Grand River near Wakpala to be expected once every 100 years is computed by using a depth of rainfall of 4.15 in. and estimating that 30% will appear as runoff:

$$\begin{aligned}\text{Rainfall of 4.15 in.} &= 0.346 \text{ ft;} \\ 0.346 \times 5,664 \times 640 &= 1,255,000 \text{ acre-ft of precipitation; and} \\ 30\% \text{ runoff} &= 377,000 \text{ acre-ft of runoff} = 190,000 \text{ sec-ft-} \\ &\quad \text{days of runoff.}\end{aligned}$$

Flood Runoff, Grand River at Blue Horse Dam Site.—The runoff of the Grand River at the Blue Horse Dam site to be expected once every 100 years is computed by using a rainfall depth of 4.40 in. and estimating that 30% will appear as runoff:

$$\begin{aligned}\text{Rainfall of 4.40 in.} &= 0.367 \text{ ft;} \\ 0.367 \times 4,085 \times 640 &= 960,000 \text{ acre-ft of precipitation; and} \\ 30\% \text{ runoff} &= 288,000 \text{ acre-ft of runoff} = 145,000 \text{ sec-ft-} \\ &\quad \text{days of runoff.}\end{aligned}$$

⁴"Storm Rainfall of Eastern United States," Miami Conservancy District, Eng. Staff, *Technical Reports*, Pt. 5, Revised Edition, 1936, Dayton, Ohio.

Unit Hydrographs.—From runoff records of the Grand River near Wakpala, hydrographs for a number of high-water periods were plotted, and the total discharge for the periods was computed and compared with the precipitation. A unit hydrograph was determined by computing the average daily percentages of the total flow during the periods. In order to determine a unit hydrograph for the Grand River at the Blue Horse Dam site, the peak flow was based on a comparison with the probable maximum peak of Grand River near Wakpala, and the base of the hydrograph was shortened to agree with the hydrographs of nearby streams having drainage areas of nearly the same size.

The discharge during a maximum flood period was computed to be as shown in Table 5. The plotting of these daily discharges shows that the peak dis-

TABLE 5.—MAXIMUM GRAND RIVER FLOOD

Day	NEAR WAKPALA		AT BLUE HORSE DAM SITE	
	Percentages	Average discharge (cu ft per sec)	Percentages	Average discharge (cu ft per sec)
1	1.2	2,280	3.0	4,350
2	6.3	12,000	16.1	23,300
3	18.8	35,700	35.8	51,900
4	27.8	52,800	24.1	35,000
5	20.1	38,200	11.9	17,260
6	12.7	24,100	5.4	7,830
7	6.3	12,000	2.4	3,430
8	3.7	7,030	1.3	1,880
9	1.9	3,610		
10	1.2	2,280		
Total	100.0	190,000	100.0	145,000

charge or flood peak would reach about 60,000 cu ft per sec for each of the locations on the river.

Comparative Method.—A study of recent floods, showing maximum peaks of record, occurring in South Dakota, Montana, and Wyoming, together with the expected Big Horn River flood from a hypothetical transposition of the 1923 Powder River storm to the Big Horn Basin, has been made. The aforementioned floods were exceptionally high and probably represent about the maximum for those streams.

The formula

$$Q = 933 A^{0.491} \dots \dots \dots (3)$$

shows the maximum flood to be expected, if such flood is of the same magnitude as shown by maximum recorded or computed hypothetical floods. Using Eq. 3, and computing for the maximum flood to be expected on the Grand River, the following quantities are obtained:

Location	Drainage area, A, in sq miles	Maximum discharge, Q, in cu ft per sec
Grand River near Wakpala.....	5,664	64,800
Grand River at Blue Horse Dam site.....	4,085	55,300

The station-year method of determining the accuracy of rainfall frequency gives the solution of the standard error of the frequency of rainfall as it occurs

at particular stations. The "dependence between stations" is an attempt to show the areal effect, or to consider contemporary rainfall at nearby stations. It does not, of course, give the error of frequency of areal storm depths.

The results of this paper can be used and are adaptable for the areal determination of the limits of storm rainfall to be expected, as it appears certain that the standard error of storm frequency as computed by the station-year method is considerably larger than the standard error of frequency of areal storm depths.

The comparison between the standard error of frequency of rainfall and the approximate error in depth of rainfall is given for Quadrangle 18-B as follows:

Years	Frequency Standard error (%)	Inches	Depth of Rainfall Error (%)
15.....	21.8	2.7.....	22.0
25.....	27.9	4.2.....	9.6
50.....	39.8	4.4.....	6.4
100.....	55.9	4.8.....	7.6

In the study of flood runoff the areal extent of storm rainfall is desired.

The flood study, from the pluvial index and the indicated standard error, was made to show its application to relatively large areas, and an allowance for the indicated error was considered, or in other words, a larger pluvial index was used to cover the possibility of the occurrence of a more intense storm.

The sizes of the probable maximum flood to be expected, as determined by using the pluvial index, adjusted for the probable error, and compared with the maximum floods of nearby streams indicate a fairly close agreement and show that the available data give a fair criterion for similar studies.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

THE GRAND CENTRAL TERMINAL IN PERSPECTIVE

Discussion

BY MESSRS. A. J. MEEHAN, AND J. P. HALLIHAN

A. J. MEEHAN,²⁶ M. Am. Soc. C. E. (by letter).^{26a}—As one who, since boyhood, has revered the Grand Central Terminal because of its inherent majesty, the writer is particularly grateful to Colonel Wilgus for presenting a comprehensive story of the inception and development of this daring enterprise.

Numerous instances in the history of this terminal have demonstrated how readily alterable it has been to meet new demands. Such adaptability has been recognized as a distinct tribute to the farsightedness of its engineering planning. The success of this huge undertaking continues to serve as an outstanding source of inspiration to other engineers who find themselves confronted with complex traffic and service problems.

The "Synopsis" and "Résumé and Conclusion" of the paper intimate that as yet there are some unsolved future problems in connection with this project. One of the remaining problems suggested itself to the writer the summer of 1940 as he gazed down upon the structure from a building on Park Avenue just south of 42d Street. In particular, it is the traffic situation on the elevated viaduct at Park Avenue and 42d Street, a block plan of which is shown in Fig. 13. This tee-shaped entrance to the viaduct surrounding the building is hazardous because of the right-angle turns and narrow roadways. At this point, the Park Avenue section of the viaduct has one lane of traffic in each direction. As one observes the traffic entering or leaving Park Avenue as it weaves into the opposing lanes in order to negotiate this turn, and likewise notes the effect upon the traffic making the continuous circuit of the terminal on the elevated driveway, he shudders to think of the accident menace and how much greater it must be when the surface is slippery.

This condition could be remedied by replacing this right-angle corner by a

NOTE.—This paper by William J. Wilgus, Hon. M. Am. Soc. C. E., was published in October, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1940, by Messrs. F. Lavis, E. R. Hill, Alonzo J. Hammond, and H. L. Ripley.

²⁶ Senior Bridge Engr., State Dept. of Public Works, Div. of Highways, Bridge Dept., Sacramento, Calif.

^{26a} Received by the Secretary December 2, 1940.

flare or curve which would provide a greater turning radius for in-bound or out-bound Park Avenue vehicles. Such a solution would require care to preserve architectural appearances and to maintain the existing under-clearance on 42d Street. However, with the great range of construction materials now available, it would be a comparatively simple matter to effect this improvement in a manner that would satisfy structural and architectural requirements.

J. P. HALLIHAN,²⁷ M. AM. SOC. C. E. (by letter).^{27a}—It is not often that the responsible designer of a facility of the importance and complexity of the Grand Central Terminal is afforded an opportunity to compare the performance with the original objectives. Colonel Wilgus has gone further in revealing how the original idea was molded into the finally accepted plan in a period when execution of the principal features involved entrance into almost uncharted fields of engineering and economics.

It is fortunate that the men of finance, of the law, of engineering, architecture, and city planning, assembled on the Commission, were also men of courage and vision, able to see that the influence of a great terminal in a great city extended far beyond its functions as a transport facility.

One of the most impressive features of the plan was the provision for taking care of the surface transportation of the future which, it is not surprising to learn, was regarded as an excessive provision by some of the members of the Commission, and was not fully carried out until some time after the completion of the terminal structure. Had this feature of the plan been surrendered, it can well be imagined what an impossible traffic situation would have been created.

In the light of present knowledge it is regrettable that the Court of Honor could not have been retained, but in the thought of that day the consequent sacrifice of building space must have made it appear as an inadmissible extravagance.

Colonel Wilgus' review of the deliberations of a commission made up of experts in their several lines, conscious of the necessity of bold pioneering in order to achieve the high purpose of getting the greatest public service within the limitations of an enterprise that must be permanently self-supporting, is a distinct service to the engineering profession.

It is safe to say that there will be no terminal development of the future that will fail to draw inspiration from the story of the Grand Central Terminal, which so effectively serves its purpose after more than a quarter century of use.

²⁷ Director, Municipal Eng. Section, WPA, Washington, D. C.

^{27a} Received by the Secretary December 23, 1940.

MODEL TESTS, BRIDGE PIER SUPPORTED
ON LONG STEEL PILES

Discussion

BY JACOB FELD, M. AM. SOC. C. E.

JACOB FELD,⁵ M. AM. SOC. C. E. (by letter).^{5a}—In a very clear manner the paper gives not only the report of tests on a model pier supported on battered column or pile supports, but also a method of designing such a structure. Of course, any design must be based upon the assumed action of the supporting piles or columns. Similarly, the construction of the test model was based on an assumed action of the supporting tubes which are called "piles."

There is a question whether the model should not be classed more properly as a pier supported on clusters of battered columns, corresponding to a possible construction for a very high viaduct. The paper states that in the prototype piles 200 ft long were used, but these piles were 30-in. shells, $\frac{7}{16}$ in. thick. To drive piles of that type for a depth of 200 ft, it is almost a self-evident conclusion that the material encountered must be very soft. However, if the material has no resistance, and therefore provides no skin friction, the question can be raised whether each of the piles or tubes can be considered safe as an individual column 30 in. in diameter and 200 ft long between bracing levels.

If the material is as soft as is stated, then considerable difficulty in sinking caissons to such depths is eliminated, and one is led to question whether a caisson design might not have been more economical. Perhaps the authors, in their concluding discussion, can advise whether a comparative study was made between a caisson design and the type shown.

By using a model ratio of $\frac{1}{40}$, a test specimen which was still of considerable height was used. However, although there can be no question as to the qualitative results, the final conclusion that under the assumed loadings a maximum lateral deflection of 9.4 in. and a maximum longitudinal deflection of 6.2 in. will result in the actual structure must be taken with some question.

In the actual completed structure, it would have been possible to measure the deflection of the pier upon imposing known horizontal pulls, not neces-

NOTE.—This paper by Thomas F. Comber, Jr., M. Am. Soc. C. E., and John M. Coan, Jr., Jun. Am. Soc. C. E., was published in June, 1940, *Proceedings*.

⁵ Cons. Engr., New York, N. Y.

^{5a} Received by the Secretary December 17, 1940.

sarily as large as the maximum expected and, in this way, provide a complete check on test results.

As it is also concluded by the authors, from the test results, that the moments and the deflections for combination loading do not differ appreciably from the sum of the individual effects similarly determined, the structure is apparently acting as an elastic body. Therefore, the determination of the deflection for smaller loads than the maximum expected can be used safely as a guide for the determination of maximum deflections under maximum loads.

The most serious objection to the model tests is the assumption of no restraint of the material through which the piles were driven. As stated previously, such an assumption is equivalent to a statement that these piles are columns which had a ratio of diameter to length of 80—considerably more than any unbraced columns normally used in construction design. Under the action of horizontal forces, an unrestrained pile would act as a column very closely approximated by a column rigidly held at the top and pin-connected at the bottom. In the model this condition was approximated closely, and the deflection curves are consistent with such an assumption.

For a pile of this length, with restraint due to skin friction and reaction of the soil in which the lower ends are embedded, the deflection curve would be somewhat different, the point of contraflexure would be higher, and the affected unsupported height would be considerably less. It is safe to assume that the resulting stresses would be lower than those indicated by the tests. However, the test cannot be considered as a complete solution of the action of piles arranged as shown in Fig. 1 when driven into a resisting medium.

In the usual case of piles driven into soils, each cluster would act as a portal under either lateral or longitudinal loads, and there would be a redistribution of the resultant moments and deflections to equalize, more closely, the reactions of the individual piles in each cluster.

Although, as shown by the authors, the assumptions made in the first method for computing moments and deflections from an elastic curve are not as accurate, as far as determining resulting reactions, as those in the second method, which actually provides for measuring strains on opposite sides of piles, the closeness of the results is valuable for decreasing the work in future tests of similar structures. There can be no question at all as to the accuracy of the results based upon the measurement of actual strains, as long as the structure remains within the elastic limit of the materials.

The procedure outlined in analyzing the test data can be used directly in designing similar structures for determining the resultant reactions, in each pile, from vertical as well as horizontal loads.

In the closing discussion it is hoped that the authors will make available any records of pile-driving resistances from the actual construction, so that an appraisal can be made, even if only qualitatively, of the assumption that there is no skin friction along the face of the piles and no restraint to bending of the embedded portions.

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DISCUSSIONS

MAXIMUM PROBABLE FLOODS ON PENNSYLVANIA STREAMS

Discussion

BY C. S. JARVIS, M. AM. SOC. C. E.

C. S. JARVIS, ¹⁴ M. AM. SOC. C. E. (by letter).^{14a}—The author has made a painstaking, logical approach toward acceptable evaluations of both flood volumes and their associated peak discharges, presumably for drainage areas ranging from 100 to 6,000 sq miles, but in practice readily adaptable to include much of the field outside those prescribed limits. For example, comparable values for 50 sq miles or for 10,000 sq miles may be readily derived by comparison and extrapolation, if the estimates for the prescribed range of 100 to 6,000 sq miles are well founded.

So many of the basic principles governing flood occurrence, magnitude, and probability have been utilized to such advantage in the author's presentation as to preclude challenge on details unless they are vital. One such exception must be noted, however, with respect to the statement (see heading "General") to the effect that these watershed factors (size and shape, location, imperviousness, slope, pattern, and nature of stream channels) remain the same for every flood on the watershed. Although the statement is accurate as to the first three factors—namely, size, shape, and location—the others are certainly subject to considerable alteration to conform with changes in land use, type and amount of vegetal cover, extent and degree of soil erosion, and the effects of corrective surface treatment designed (1) to lengthen the path traversed by surface runoff, (2) to reduce the gradients in the same proportion for the elementary channels, and (3) to alter the drainage pattern on the catchment area, before the runoff has concentrated into permanent channels of considerable capacity. As a result of changes in these three factors, a marked influence may be exerted on the imperviousness or, perhaps better, on the infiltration capacity of such drainage areas.

Any one versed in the practice of irrigation can appreciate the significance

NOTE.—This paper by Charles F. Ruff, M. Am. Soc. C. E., was published in September, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1940, by Messrs. Joseph L. Benson, H. Alden Foster, and Edgar E. Foster.

¹⁴ Hydr. Engr., SCS, Dept. of Agriculture, Washington, D. C.

^{14a} Received by the Secretary December 20, 1940.

of the foregoing comments by reference to the behavior of canal water as applied to newly reclaimed areas. If the water is allowed to follow the natural swales which may trend diagonally across a field, it will tend to erode the swale into a definite channel to permit flow on the steepest gradient, and to waste most of the water beyond the border of the field. However, the construction of guiding levees, terraces, straw or other dams, and regulating devices at suitable intervals will insure an equitable distribution into every furrow, flowing on gentle gradients for several hours, and retention of practically the entire volume of irrigation supply with little, if any, wastage of either soil or water.

Practical forestry, range management, and soil conservation measures today have as their objective an approach to the aforementioned condition for the conservation of available water, along with soil structure and fertility. To the extent that these conservation measures, including flood storage and detention, become effective and helpful in establishing permanent vegetal cover, either as forage or other perennial growth, a degree of protection and regulation is achieved, as has been demonstrated by numerous plots, fields, and small watersheds, and occasionally by larger units both in the United States and abroad. These surface treatments more or less offset the obviously negative effect of such disturbances of land surface as have attended man's occupation and infringements upon natural storage and overflow areas. Obviously, conservation measures have their limitations; but these have seldom been attained or closely approached.

It is reassuring to note the author's frank estimate of the intangibles connected with maximum flood estimates (see heading "General") and the cryptic expression, "The only certainty in regard to any future storm is that it will not exactly duplicate those of the past nor, for that matter, those developed herein." Such statements clarify the objective served by many ingenious devices and avenues of approach as used by the author.

The valley at 810 miles and the hump at 1,250 miles in Fig. 4 may appear somewhat irregular, but to the writer they seem quite reasonable as denoting, respectively, minimum storm depths due to remoteness from source of supply, and increased precipitation due to surmounting the Allegheny barrier ridges.

The map of storm paths in Table 1 seems to neglect the scope of the May-June, 1889, storm on the eastward slopes of the Appalachian Range; yet it is known that unprecedented floods of record, associated with the Johnstown, Pa., disaster, occurred on both the Potomac and the Susquehanna basins at that time, which continued as the record maxima until the March, 1936, floods established new records. Indeed, it may be impossible to depict a storm path satisfactorily by a single line, indicating merely the course followed by its center, when the disturbance is general over a width of several hundred miles, astride a prominent mountain range such as the Appalachian system. Should one not add some symbols to indicate width and scope of storm, as exemplified in Fig. 20? The added scale or index of width could be provided as shown at the northern terminus of storm path A, or at the southern ends of B, C, and D to indicate 200, 500, and 300-mile widths, respectively, with each added line representing 100 miles.

If the standard flood peaks vary as the 0.7 power of the drainage area (see heading "Development of the Standard Floods: The Standard Flood"), and the standard floods vary as the 0.3 power, then the mean variation is evidently in accordance with the 0.5 power. The writer's entire experience, confirmed

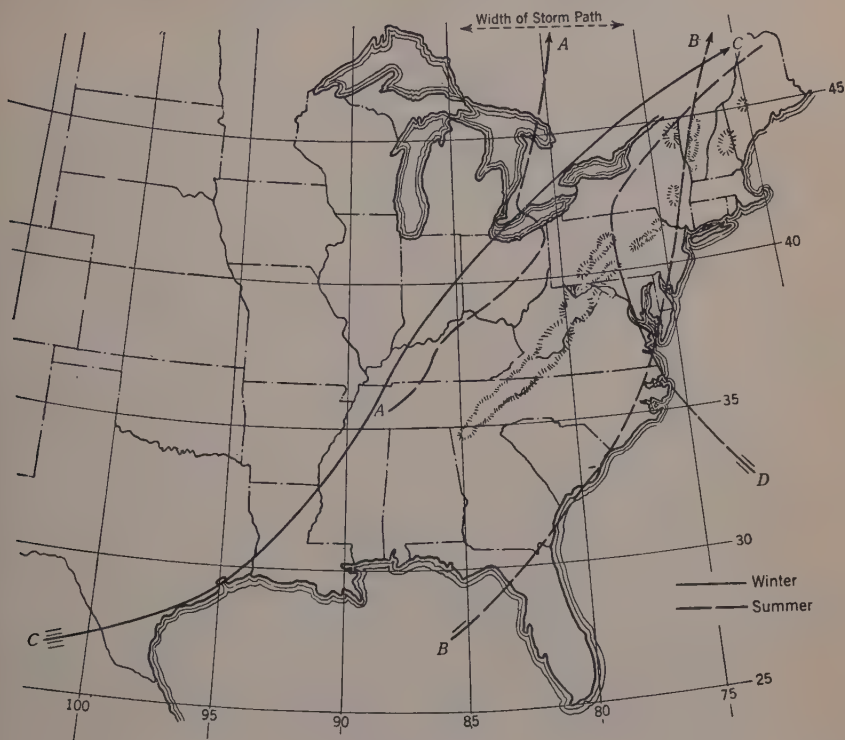


FIG. 20.—MAXIMUM PROBABLE FLOODS ON PENNSYLVANIA STREAMS

by the graphs of this paper, is opposed to the 0.3 power relationship. In general, the slope of the envelope curve between 100 and 1,000 sq miles should be somewhat less than 0.5, rather than greater, indicating that the power is greater than 0.5, inasmuch as the relationship is inverse or reciprocal; that is, the slope measures the departure from unity, and, therefore, the greater the slope, the smaller is the fractional power. Furthermore, from the context following and the graph of Fig. 18, it is doubtful if any segment of the curve represents 0.3 power; it is more nearly 0.7 power. Quoting from the paper (see heading "Comparison with Flood Records"), furthermore: "Considering only the flood points, it appears that a steeper line, perhaps one represented by the formula [Eq. 6] would fit the data better."

It is difficult to follow the steps and to accept the results given in the paper preceding Table 5 in connection with storm frequency, such as

"* * * The total area covered by the Atlantic Summer storms is about 400,000 sq miles. Of this area, 13,000 sq miles lie in the Lower Susque-

hanna and Delaware basins of Pennsylvania subject to such storms. Thus, there is about one chance in thirty that a storm occurring somewhere in the entire area will occur in the state. If one storm in thirty occurs in the state, the average interval between such storms will be 30 times 17 years or 510 years. The Ohio storms, both summer and winter, covered an area of roughly 780,000 sq miles. On the same basis the approximate frequency of the storms within Pennsylvania is as shown in Table 5."

The foregoing quotation indicates that only average weight is accorded the Pennsylvania areas, when such factors as topography, elevation, and convergence of usual storm paths seem to increase the storm rainfall and runoff potentialities far above average. Likewise, no allowance seems to have been given the storm disturbances that occasionally traversed their way between or among the widely spaced rainfall stations of early periods. There are abundant indications that many storm occurrences have never been included in the record, because their centers happened to avoid the established meteorological stations, and only the fringes were officially observed. Taking such factors into consideration, the writer would favor revision of some of the computed average intervals between storms in Pennsylvania to approximately one half the values shown in Table 5.

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DISCUSSIONS

MASONRY DAMS

A SYMPOSIUM

Discussion

BY MESSRS. I. NELIDOV, AND JAMES B. HAYS

I. NELIDOV,¹¹⁵ M. AM. SOC. C. E. (by letter).^{115a}—Referring to “Basic Design Assumptions” so fully presented by Messrs. Houk and Keener, the writer wishes to discuss the following features of the design of high dams: The effect of Poisson’s ratio on the stresses in straight and curved gravity and buttress dams, the validity of the shear-sliding ratio, and the major stress problems involved in raising a dam. The authors limited themselves primarily to discussion of dams of solid type. However, with buttress dams advancing into the high-dam class the writer saw fit to include them also in the discussion. The simplest theoretical deductions were used in order not to obscure the issue. Numerical values given in the discussion have only a relative meaning.

As is known, the true unit deformations, or deformations including lateral distortions due to Poisson’s ratio, are expressed as follows:

e_x = 1/E σ_x - η (σ_y + σ_z).....(17a)

e_y = 1/E σ_y - η (σ_x + σ_z).....(17b)

and

e_z = 1/E σ_z - η (σ_x + σ_y).....(17c)

in which: σ = apparent stresses, with subscripts corresponding to three orthogonal axes; η = 0.2 = Poisson’s ratio; and E = the modulus of elasticity. The apparent stresses, σ, are determined from boundary conditions of loading, equations of equilibrium, and the compatibility equation.

NOTE.—This Symposium was published in May, 1940, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: September, 1940, by Messrs. William P. Creager, J. R. Shank, George R. Rich, Robert A. Sutherland, Ross M. Riegel, Paul Baumann, W. A. Perkins, L. J. Mensch, and Lewis H. Tuthill; October, 1940, by Messrs. F. A. Nickell, Leslie W. Stocker, Barton M. Jones, P. E. Gisiger, Joseph A. Kitts, S. O. Harper, and R. F. Blanks; November, 1940, by Messrs. Berlen C. Money-maker, A. Warren Simonds, and W. J. E. Binnie; and December, 1940, by Homer M. Hadley, Assoc. M. Am. Soc. C. E.

¹¹⁵ Senior Engr. of Hydr. Structure Design, Dept. of Water Works, State Div. of Water Resources, Sacramento, Calif.

^{115a} Received by the Secretary December 3, 1940.

It may be noted that true stresses based on unit deformations will be:¹¹⁶

$$\sigma_x' = e_x E \dots \dots \dots (18a)$$

$$\sigma_y' = e_y E \dots \dots \dots (18b)$$

and

$$\sigma_z' = e_z E \dots \dots \dots (18c)$$

The following notations are used for the apparent stresses: At the upstream face, σ_x = the horizontal stress due to arch action; σ_y = the horizontal stress due to water pressure; and σ_z = the vertical stress due to water load and own weight of the dam. The true stresses, σ_x' , σ_y' , and σ_z' , correspond to the foregoing apparent stresses. At the downstream face, σ_1 = the stress parallel with the face, acting in a vertical plane; σ_2 = the stress normal to the face; and σ_3 = the stress parallel with the face acting in a horizontal plane. The corresponding true stresses are σ_1' , σ_2' , and σ_3' .

The foregoing stresses are assumed to be equivalent to principal stresses so as to avoid introducing complication due to shears. It was assumed that there is no tension in the arch.

The effect on the stresses of an absolutely unyielding abutment was computed approximately by making $e_x = 0$ and determining σ_x or σ_3 from Eq. 17a. Curved gravity dams and buttress dams are considered.

The resulting stresses are given in Table 19, which shows that, at the

TABLE 19.—COMPUTATION OF APPARENT STRESSES
($\sigma_y = 250$; $\sigma_z = 100$; $\sigma_1 = 600$; and $\sigma_2 = 0$, for each case)

Case No.	UPSTREAM FACE				DOWNSTREAM FACE				Remarks
	σ_x	q_x'	σ_y'	σ_z'	σ_2	σ_1'	σ_2'	σ_3'	
1	100	30	210	30	0	600	-120	-120	Curved Gravity Dam:
2	0	-70	230	50	100	580	-140	-20	At crown
3	70	0	216	36	120	576	-144	0	At abutment
4	0	-70	230	50	0	600	-120	-120	Right abutment Buttress dam

upstream face, tension of 70 lb per sq in. exists in the direction of the arch, in case No. 2. This tension decreases to zero for the rigid abutment, in case No. 3. For a buttress in case No. 4, it becomes more dangerous since there is no lateral support.

At the downstream face tensions exist also at the crown and, of the two tensions, $\sigma_2' = -120$ lb per sq in. is the most dangerous since it is acting away from the face that has no lateral support. At the abutment, tension in the direction of the arch is decreased due to the rigidity of the abutment, but it is increased in the direction normal to the face. In the case of a buttress (case No. 4) tensions of $\sigma_2' = \sigma_3' = -120$ lb per sq in. exist in the directions normal to the face and to the axis of the buttress. Both these stresses are dangerous since there is no lateral support to resist them.

¹¹⁶"Mechanics of Materials," by the late Mansfield Merriman, M. Am. Soc. C. E., 11th Ed., John Wiley & Sons, New York, N. Y., p. 359.

If the tensile strength of concrete is 250 lb per sq in., the factor of safety is about: $\frac{250}{120} = 2.1$, which in itself might be regarded as sufficient provided one could rely on the uniformity of the strength of concrete. It may be noted that the factor of safety in compression for a 3,000 lb per sq in. concrete will be $\frac{3,000}{600} = 5$.

The writer believes that consideration should be given to the effect of Poisson's ratio when designing high concrete dams and especially those of the buttress type.

In commenting on the shear-sliding factor in computing stability of a gravity dam against horizontal forces, it may be noted that sliding cannot occur simultaneously with shear, but will begin only after the shearing failure has occurred. Consequently, it appears that only shearing resistance should be introduced in computing stability. At the same time, since sliding resistance is ordinarily smaller than shearing resistance, the minimum factor of safety should be based on sliding alone, and the maximum factor on shearing alone. Of course, this excludes the case in which the excavation profile is so rugged that no sliding can possibly occur. A comparison should be made in each case between sliding and shearing resistance. Assuming an average compressive stress of 100 lb per sq in., an area of the base equal to 200 sq ft, the friction coefficient equal to 1, and without going into the detail of computing shearing resistance based on actual stress distribution, the sliding resistance will be: $100 \times 144 \times 200 \times 1 = 2,880,000$ lb. The shearing resistance based on a shear of 200 lb per sq in. will be: $200 \times 144 \times 200 = 5,760,000$ lb. This indicates that sliding resistance controls in this particular example.

In recent years another problem has become connected with the design of high dams—the problem of either raising the existing dam, or providing for a future increase in height. The actual cases in which the heights of dams have been increased are not numerous. Among the more important are: Assuan Dam, in Egypt; Huntington Lake Dam, Sweetwater Dam, O'Shaughnessy Dam, and Alpine Dam, all in California. As might be anticipated, there are many design and construction questions involved in adding to the height of a dam. The method used at O'Shaughnessy and Alpine dams consisted principally of adding a slab of concrete to the downstream face of the existing dam, thus forming an inclined joint between the old and new concrete. In order that the dam might be monolithic this joint should be capable of transmitting the imposed stresses. In the O'Shaughnessy and Alpine dams the downstream faces of the existing dams were stepped (3.5 run and 5.0 rise). These steps were roughened by chipping.

In Fig. 29(a) the elementary cube is shown inscribed into the steps, with its proportions referred to the dimensions of the steps, and forces acting on its side. Thus p and p' denote normal stresses; q = shearing stresses; and f = principal stresses. Fig. 29(b) is Mohr's circle of stress; and a force diagram is shown in Fig. 29(c).

In case both vertical and horizontal steps are entirely ineffective in carrying shear, the forces $q_v \times 0.520$ and $q_h \times 0.364$ should be replaced by a force $A B = 98$ lb per lin in. of length of the dam, acting at an angle 35° with the

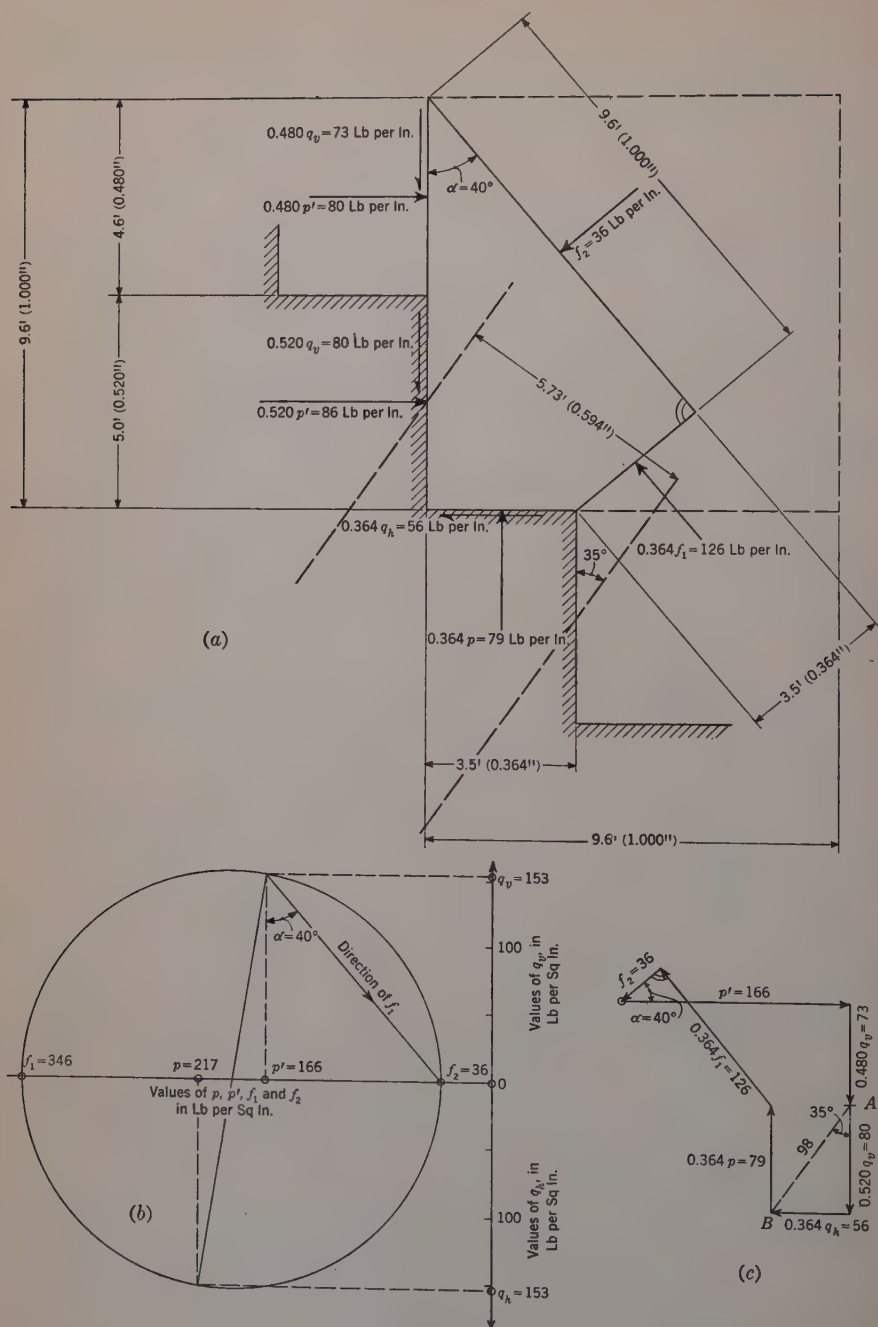


FIG. 29

vertical. Since the steel will be placed at the middle of each rise of the step the area on which this force acts is 5.73 ft or, reduced to the proportion of the elementary cube, 0.594 in.

The unit tensile stress in concrete developed in the absence of shear resistance of steps is then: $f = \frac{98}{0.594} = 165$ lb per sq in. The area of concrete per each step and per linear foot of the length of dam is: $A = 5.73 \times 1 \times 144 = 825$ sq in. The total tensile stress for this area will be: $T = 166 \times 825 = 136,000$ lb.

The area of steel required, with an allowable unit stress of 16,000 lb per sq in., will be: $A_s = \frac{130,000}{16,000} = 8.5$ sq in. The percentage of steel is, then: $\frac{8.5}{825} = 1.03\%$.

Selecting $1\frac{1}{4}$ -in. square bars it will require $\frac{8.5}{1.56} = 5.5$ bars per step and per foot of the dam. Actually, surfaces of steps are able to take friction and shear. The ultimate shearing strength of the joint along the steps can be taken at 600 lb per sq in. which, with a factor of safety of 4, will make the allowable stress 150 lb per sq in.

The resisting shearing forces corresponding to this stress will be: $150 \times 0.520 = 78$ lb per sq in. and $150 \times 0.364 = 55$ lb per sq in.; and the stress in steel will be $\sqrt{(80 - 78)^2 + (56 - 55)^2} = 2.2$ lb per sq in. This will require the area of steel $A_s = 8.5 \times \frac{2.2}{98} = 0.19$ sq in., or about $5.5 \times \frac{2.2}{98} = 0.12$ bars per lin ft of the dam. The angle of inclination of bars with vertical is 35° .

The ultimate frictional resistance with compressive stresses about 200 lb per sq in. may be assumed to be about 4, so that, with the factor of safety of 4, the allowable friction coefficient will be 1. This will produce the resisting frictional forces equal to $166 \times 0.52 \times 1 = 86$ lb per sq in. and $217 \times 0.364 \times 1 = 79$ lb per sq in. Since these forces are greater than the acting shearing forces, no steel is needed.

Another consideration enters the design of steel bars—it is the grouting pressure due to grout inserted in the joint after new concrete is poured, in order to make the joint tight. Assuming a grouting pressure of 50 lb per sq in., the total tensile force per step and per linear foot of the dam will be: $T = 50 \times 825 = 41,250$ lb. The vertical depth of the new concrete over the joint is $h = 113$ ft and its unit weight is 150 lb per cu ft. The component of its weight resisting the grouting pressure is: $P = 150 \times 113 \times 3.5 \times 1 \times 0.574 = 34,100$ lb. The remaining part of the grouting pressure, $41,250 - 34,100 = 7,150$ lb, must be taken by steel. If steel is placed at an angle of 35° with the vertical, the axial component of this force will be: $7,150 \times \cos 20^\circ = 6,700$ lb, and the transverse component will be: $7,150 \times \sin 20^\circ = 2,440$ lb.

The area of steel for tension is then: $A_s = \frac{6,700}{16,000} = 0.42$ sq in. per lin ft of the dam, or $\frac{0.42}{8.5} \times 5.5 = 0.27$ bar. The area of bars required for shear is

then: $A_s = \frac{2,440}{12,000} = 0.20$ sq in. per lin ft of the dam. The total area of steel required for carrying the water load and resisting grouting pressure will be: $A_s = 0.19 + 0.42 = 0.61$ sq in. or $0.12 + 0.27 = 0.39$ bar per lin ft of dam. By taking $1\frac{1}{4}$ -in. square bars and spacing them on 2.5-ft centers, the desired area is obtained.

JAMES B. HAYS,¹¹⁷ M. Am. Soc. C. E. (by letter).^{117a}—To the statement about core drilling interpretations in the excellent paper by Messrs. Paul and Jacobs, the writer would like to add particular emphasis. The type of equipment best suited for maximum recovery should be used. Hydraulic-feed diamond drills will recover more of the softer forms of rock than the screw-feed machines. Continuous observation of the drilling, such as changes in color of the return drill water, the pressure or rate of feed on the drill itself, together with notes on the length of rods or depth of hole at each change, will be of considerable aid in preparing an accurate log of the hole when less than a complete recovery of core is brought up in the core barrel.

On the subject of drainage the writer would like to call attention to the fact that, in many instances in the past, drainage holes have been drilled into rock too close to a grouted cutoff curtain so that they penetrated rock that had been tightly grouted and did not function as drains. It is often the case that grout and drain holes are drilled from the same gallery. If this is done, the grout holes should be inclined in an upstream direction and the drain holes sloped downstream to clear the previously grouted zone or curtain.

The writer heartily agrees with Mr. Tyler in his paper on concrete control and would like to add some points on cracking from personal observations. Foundation cracks can be largely eliminated by placing concrete in thin layers on the rock and allowing time for heat loss. Reinforcement has also been used over sharp breaks in rock ledges. Horizontal joints also should be made at such points. Other cracks are caused by improper design of concrete sections. However, as Mr. Tyler states, 75% or more of the cracking is due to temperature effects.

A few years ago the writer observed that where forms were removed from mass concrete sections twenty-four hours after pouring, and water curing started immediately, the cracking was noticeably less than where forms were left in place for two to five days. Douglas McHenry prepared some computations along this line (unpublished) and came to the conclusion that the concrete would stand a far greater temperature drop without cracking at twenty-four hours than it would from two to five days later. Beyond this time conditions slowly improve so that after twelve or fifteen days the forms may be removed with about as much danger of causing the concrete to crack as if removed in twenty-four hours. It is imperative that the concrete be kept moist at all times. When forms are removed in twenty-four hours, water curing should be started immediately and should be maintained continuously for as long a period as possible. When wood forms are left in place for a long period, cracks often will

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^{117a} Received by the Secretary December 16, 1940.

be found if the concrete has dried out. The writer's experience along the foregoing lines covers two projects in which limestone coarse aggregate was used and the "pours" were in 10-ft lifts. Cracks have not been eliminated completely, but those few that have occurred are very fine and limited in extent.

When forms are left in place for a period of several days or longer, absorbent form-lining materials will keep the surface of the concrete moist and will prevent cracks caused by drying. Concrete surfaces poured against absorbent form lining resist the absorption of water to a high degree, and indications from observation are that the dense surface also loses less water from evaporation. Observation on concrete placed against an absorbent form lining denotes that almost no cracks occur, compared to a face placed against a good oiled-wood form. Absorbent form lining has been used at Kentucky Dam for several months and the results, so far, are very pleasing.

The writer would add three other items to the list of seven of the procedures tending toward crack elimination, as follows: (8) Early removal of forms and the immediate and continued application of curing water; (9) use of absorbent form-lining materials; and (10) selection of proper aggregate, if available.

Mr. Steele has given a thorough analysis of the problem of construction joints. The writer would like to add a few notes from his experience in grouting contraction joints, gained while working under Mr. Steele when considerable experimental work was being done along this line. The work at Boulder Dam was described¹¹⁸ by the writer in 1937. Earlier methods used by the writer on the Calderwood Dam were described¹¹⁹ in 1933.

Experience indicates that the use of horizontal grout stops should be avoided and the grouting layout arranged so that all joints can be grouted simultaneously from bottom to top. This is not always possible, but this fact need not prevent obtaining a satisfactory job. The elimination of the horizontal grout stops will permit the regrouting of lower lifts when the upper sections of the joints are opened during grouting. Horizontal grout stops placed above the outlet headers of the grouting system leave a space where thin, foamy grout accumulates and cannot be drained off. Horizontal stops will be necessary at the top of the dam, as well as under and over galleries. Since these stops are difficult to install, the writer prefers to follow a zigzag or sloping pattern. In this way there is only a short section of the seal covered by the concrete at any one time, and if the seal becomes bent down it will be indicated above the surface. There is also less chance of getting honeycomb concrete just under the seal. High, thin, arch dams must be grouted with extreme care to avoid excessive upstream deflections that might cause damage to the structure.

In comparing grouted joints with concrete filled slots, there is some advantage in favor of the grout, in that open cracks in the joint faces of the concrete will be filled.

¹¹⁸ *Civil Engineering*, February, 1937, p. 126.

¹¹⁹ *Loc. cit.*, November, 1933, p. 606.

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DISCUSSIONS

FOUNDATION EXPERIENCES, TENNESSEE VALLEY AUTHORITY

A SYMPOSIUM

Discussion

BY BARTON M. JONES, M. AM. SOC. C. E.

BARTON M. JONES,³⁰ M. AM. SOC. C. E. (by letter).^{30a}—The subject of Mr. Lewis' excellent paper is of fast growing importance along with improvements in the methods and procedure of treating dam foundations. As time goes on, dams will be constructed upon foundations less perfect than those now available. Practices developed to make the less perfect sites satisfactory at costs that are feasible and to obviate the need for removing large masses of somewhat inferior rock and replacement with costly concrete will be of inestimable value.

The paper applies mostly to the horizontally bedded type of formation. The Norris foundation, of this type, was extremely open and the filling of the seams and cavities was possibly more necessary from the standpoint of bearing than from watertightness, although both were of prime importance. The paper does not refer to the inadequate bearing resistance.

Mr. Lewis is to be commended for bringing together and describing the many and varied operations and pieces of equipment used in the foundation treatment and in mentioning what may seem to be some of the minor details that are often omitted in papers but which in reality supply extremely useful information to those having such work in hand. Also, he should be commended for the frank occasional admission that some other method or device, if it had been used, would have been better.

The effectiveness and moderate cost of the treatment of the Norris foundation reflect the continued care and study given to all situations as they arose and in formulating plans that would be most suitable in each case. The dam is practically bottle-tight. With reference to tightness against leakage, it

NOTE.—This Symposium was published in March, 1940, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: May, 1940, by Messrs. George K. Leonard, and F. B. Marsh; June, 1940, by Messrs. Berlen C. Moneymaker, R. F. Walter, William F. Prouty, Jacob Feld, and A. Warren Simonds; and September, 1940, by Messrs. V. L. Minear, and C. E. Blee.

³⁰ Cons. Design Engr., TVA, Knoxville, Tenn.

^{30a} Received by the Secretary November 12, 1940.

should be explained that evidence indicates that the largest item of water from drain holes mentioned in the paper, namely 0.55 cu ft per sec from under the spillway apron (see heading "Curtain Grouting: Leakage"), is supplied mostly from sources in the sidehills and abutments rather than from the reservoir.

The paper does not explain that the adopted system of patterns and sequence used in drilling the grout holes was effective as well from the standpoint of drilling progress and economy as for both the washing and grouting operations.

The parallel tunneling on one large seam in the east or left abutment might warrant some amplification beyond the description given by Mr. Lewis to reveal other important features. To avoid weakening the overlying slab, it was left intact. It formed the roofs of the several parallel tunnels which were excavated entirely downward from the seam, in the under-side layer of rock. The 20-ft parallel spacing of the tunnels allowed a thorough cleaning out and washing of the seam between tunnels under dry accessible conditions and refilling with dry packed cement mortar where the opening was sufficient to permit it. Very satisfactory and rapid concreting of the tunnels was done through 36-in. holes at their far ends, drilled from the surface, and through which concrete was dumped in 3-yd batches with considerable impact. Heavy bulkhead forms were built to retain the concrete when it reached the portals.

It may be noted that all grouting was done at relatively low pressures considering the height of the dam. The low pressures proved entirely adequate. The aim was to use pressures as high as possible, but limitations were brought about by measurable effects detected by upheaval gages installed for measuring vertical movement of the rock at its surface, as explained in the paper. Under the completed dam a pressure of 150 lb per sq in. was frequently found to be too high. The upheaval gages (heading "Curtain Grouting: Upheaval Gages") were first proposed by the writer and perfected by the engineering force at Norris Dam. A subsequent design supplied by the Bureau of Reclamation was not used because dial indicators made the device too delicate for use around heavy construction operations.

It is believed that core drills making holes large enough for a man to enter and inspect the undisturbed formation—such as the 36-in. shot drills—were used first at Norris Dam where the immediate result was a reduction in foundation stripping with important savings in construction cost. It might be suggested that grouting a single small hole in the center of a proposed large hole, instead of four holes outside, would ordinarily close the surrounding water-bearing seams sufficiently to permit nearly dry drilling.

The paper by Mr. Lewis might have directed more attention to the uncertainty of indications as to the amount of seams and cavities as shown by cores from holes of small drills, such as $2\frac{1}{16}$ in. for example. Also, it might have emphasized the variation of indications due to causes such as slow and steady drilling versus careless drilling, the use of a double versus single barrel, and the frequent pulling of cores. To the use of the double barrel, Mr. Lewis ascribes a core-loss indication of 25% reduced to 6% in the same rock. Small holes generally, or invariably, exaggerate the defects of the formation drilled.

The rock flour, mentioned by Mr. Lewis as an admixture to the grout used in the rim treatment, was a by-product. The dolomite sand, produced by hammer mills, was washed and the wash water passed through an improvised classifier which divided the suspended particles at a size of about 200-mesh. The material coarser than 200-mesh was used to balance the grading of the sand used for concrete. Material smaller than 200-mesh was stored for future use as fertilizer, and it is this material (called "rock flour" by Mr. Lewis) that was used in some of the grout.

Correction for *Transactions*: March, 1940, *Proceedings*, page 406, delete the sentence beginning on line 6.